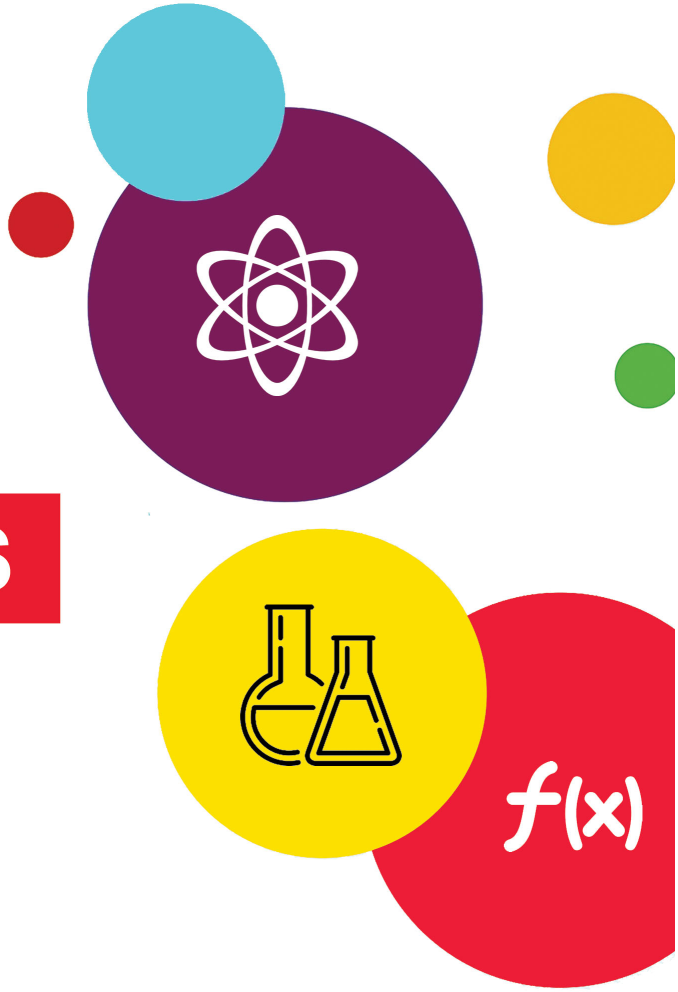
STUDY MATERIAL

For JEE-MAIN

PHYSICS

CHEMISTRY

MATHEMATICS



CP PUBLICATION

CAREER POINT

Study Material for JEE Main preparation

Prepared by Career Point Kota Experts

CONTENTS OF THE PACKAGE AT A GLANCE

PHYSICS

Class 11

Module-1

- Unit, Dimension & Error
- Vector
- Motion in one dimension
- Projectile motion
- Newton's laws of motion & friction

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- Work, Power, Energy & Conservation Law
- GRAVITATION
- Simple Harmonic Motion
- PROPERTIES OF MATTER
- FLUID MECHANICS

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- Calorimetry
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Class 11

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- Chemical Energetics
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Class 12

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MATHEMATICS

Class 11

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Note to the Students

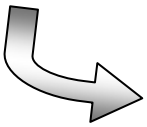
Career Point offers this must have Study Package in Physics, Chemistry and Mathematics to meet the complete curriculum needs of engineering aspirants. The set comprises of 18 books: **Physics** - set of 3 books for class 11 and set of 3 books for Class 12; **Chemistry** - set of 3 books for class 11 and set of 3 books for Class 12 and **Mathematics** - set of 3 books for class 11 and set of 3 books for Class 12. The set caters to the different requirements of students in classes XI and XII. It offers complete and systematic coverage of **JEE Main** syllabi and aims to provide firm foundation in learning and develop competitive edge in preparation of the JEE and other engineering entrance examinations.

COMPONENTS OF EACH CHAPTER

These books are designed with an engaging and preparation-focused pedagogy and offer a perfect balance of conceptual learning and problem solving skills.

Theory & Concepts

Each chapter consists of high quality theory that covers all the topics, sub-topics and concepts of JEE syllabus.

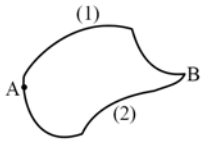


Motion in One Dimension

KEY CONCEPT

1. Distance

Distance is the actual length of the path. It is the characteristic property of any path i.e. path is always associated when we consider distance between two positions. Distance between A and B while moving through path (1) may or may not be equal to the distance between A and B while moving through path (2).



(a) It is a scalar quantity

(b) Dimension : $[M^0L^1T^0]$

(c) Unit : In C.G.S. centimeter (cm), In M.K.S. (m)

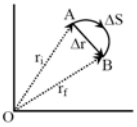
2. Displacement

Displacement of a particle is a position vector of its final position w.r.t. initial position.

Displacement =

$\overrightarrow{AB} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\hat{k}$

It is the characteristic property of any point i.e. depends only on final and initial positions.



In the above figure distance travelled is ΔS, while displacement is $\overrightarrow{\Delta r} = \vec{r}_f - \vec{r}_i$

3.1 Regarding distance and displacement it is worth noting that :

(i) Distance is scalar, while displacement is vector both having same dimensions [L] and same SI unit metre.

(ii) The magnitude of displacement is equal to minimum possible distance so,

Distance ≥ |Displacement|

(iii) For motion between two points displacement is single valued, while distance depends on actual path and so can have many values.

(iv) For a moving particle distance can never decreases with time while displacement can. Decrease in displacement means body is moving towards the initial position.

Important Points

This part contains important concepts & formulas of chapter at one place in short manner, So that student can revise all these in short time.

Important Points

- If body starts from rest or falls freely or is dropped then, u = 0
- If the body is thrown upwards then it will rise until its vertical velocity becomes zero. Maximum height attained is $h = u^2/2g$.
- If air resistance is negligible then the time of the rise is equal to time of fall and each is equal to t = u/g.
- The body returns to the starting point with the same speed with which it was thrown.
- The straight line inclined to time axis in x-t graph represents constant velocity.

Solved Examples (JEE Main)

To understand the application of concepts, there is a solved example section. It contains large variety of all types of solved examples with explaination to ensure understanding the application of concepts.

SOLVED EXAMPLES

Ex.1 The velocity acquired by a body moving with uniform acceleration is 20 meter/second in first 2 seconds and 40 m/sec in first 4 sec. The initial velocity will be -

(A) 0 m/sec (B) 40 m/sec

(C) 20 m/sec (D) None

Sol. $\text{Acceleration} = \dfrac{\text{Change in velocity}}{\text{Time}}$

$$= \frac{40-20}{4-2} = 10 \text{ m/sec}^2$$

From the relation, $v = u + at$

$\Rightarrow 20 = u + 10 \times 2 \Rightarrow u = 0$ m/sec

Hence correct answer is (A).

Ex.2 A body moves along the sides AB, BC and CD of a square of side 10 meter with velocity of constant magnitude 3 meter/sec. Its average velocity will be-

(A) 3 m/sec (B) 0.87 m/sec

(C) 1.33 m/sec (D) None

Sol. Average velocity of the body

$$= \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{AD}{(AB+BC+CD)/v}$$

$$= \frac{10}{30/3} = 1 \text{ m/sec}$$

Ex.4 A point travelling along a straight line traverse one third the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. The mean velocity of the point averaged over the whole time of motion will be -

(A) $\dfrac{v_0(v_1+v_2)}{3(v_1+v_2+v_3)}$ (B) $\dfrac{3v_0(v_1+v_2)}{v_1+v_2+v_3}$

(C) $\dfrac{v_0(v_1+v_2)}{v_1+v_2+4v_3}$ (D) $\dfrac{3v_0(v_1+v_2)}{v_1+v_2+4v_0}$

Sol. Let s be the total distance . Let (s/3) distance be covered in time t_1 while the remaining distance (2s/3) in time t_2 second.

Now $\left(\dfrac{s}{3}\right) = v_0 t_1$ or $t_1 = \dfrac{s}{3v_0}$ .....(1)

and $\left(\dfrac{2s}{3}\right) = v_1\left(\dfrac{t_2}{2}\right) + v_2\left(\dfrac{t_2}{2}\right)$

or $t_2 = \dfrac{4s}{3(v_1+v_2)}$ .....(2)

Average velocity $= \dfrac{s}{t_1+t_2} = \dfrac{s}{\frac{s}{3v_0} + \frac{4s}{(v_1+v_2)}}$

$$= \frac{3v_0(v_1+v_2)}{v_1+v_2+4v_0}$$

Hence correct answer is (D)

Practice Exercises

Exercise Level - 1 : It contains objective questions with single correct choice to ensure sufficient practice to accutrately appply formule and concepts.

Exercise Level - 2 : It contains single objective type questions with moderate difficulty level to enahcne the conceptual and application level of the student.

Exercise Level - 3 : This execrcise contains AIEEE, JEE Main off lines & online exam Questions from old years to upto recent year.

EXERCISE - 3

Old Examination Questions [AIEEE/JEE Main]

Q.1 A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is – **[AIEEE-2002]**

(A) 16 J (B) 8 J
(C) 32 J (D) 24 J

Q.2 A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is – **[AIEEE-2003]**

(A) 18.75 N-m (B) 25.00 N-m
(C) 6.25 N-m (D) 12.50 N-m

Q.3 A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total of the chain is 4 kg. What is the work done in pulling the entire chain on the table – **[AIEEE-2004]**

(A) 7.2 J (B) 3.6 J
(C) 120 J (D) 1200 J

Q.4 A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\vec{i} - \vec{j})$ m. The work done on the particle in joules is – **[AIEEE-2004]**

(A) – 7 (B) + 7
(C) + 10 (D) + 13

Q.5 A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by- **[AIEEE-2005]**

(A) $\frac{mv^2}{T^2}.t$ (B) $\frac{mv^2}{T^2}.t^2$
(C) $\frac{1}{2}\frac{mv^2}{T^2}.t$ (D) $\frac{1}{2}\frac{mv^2}{T^2}.t^2$

Q.51 A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is - **[JEE Main - 2018]**

(A) $-\frac{k}{4a^2}$ (B) $\frac{k}{2a^2}$
(C) zero (D) $-\frac{3}{2}\frac{k}{a^2}$

Q.52 A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first second after the start of the motion is - **[JEE-Main Online-2018]**

(A) $\frac{1}{8}$ ma^4t^2 (B) $4ma^4t^2$
(C) $8ma^4t^2$ (D) $\frac{1}{4}$ ma^4t^2

Q.53 In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % grater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is - **[JEE Main - 2018]**

(A) $\frac{v_0}{4}$ (B) $\sqrt{2}\,v_0$ (C) $\frac{v_0}{2}$ (D) $\frac{v_0}{\sqrt{2}}$

Q.54 A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass of unknown particle is - **[JEE-Main Online-2018]**

(A) $\frac{m}{\sqrt{3}}$ (B) $\frac{m}{2}$ (C) 2m (D) m

Exercise Level - 4 : This exercise contains IIT, JEE advanced exam previous year questions from old years to recent year.

EXERCISE - 4

Old Examination Questions [IIT JEE Advanced]

Q.1 An elastic spring of unstretched length L and force constant K is stretched by a small length x. it is further stretched by another small length y. The work done in the second stretching is – **[IIT - 1994]**

(A) $\frac{1}{2}ky^2$ (B) $\frac{1}{2}k(x^2+y^2)$

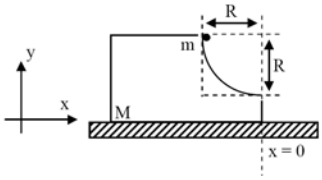
(C) $\frac{1}{2}k(x+y)^2$ (D) $\frac{1}{2}ky(2x+y)$

Q.5 A wind -powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power out put will be proportional to – **[IIT - 2000]**

(A) v (B) v^2

(C) v^3 (D) v^4

Q.25 A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct ? **[JEE Advanced-2017]**



Q.27 A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimension. Which of the following statements is (are) true?

MCQ **[JEE-Advanced-2018]**

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increases linearly with time

(D) The force is conservative

Answer key

Ankswer key is provided at the end of the exercise sheets.

ANSWER KEY

EXERCISE - 1

1. (A)	**2.** (C)	**3.** (B)	**4.** (D)	**5.** (A)	**6.** (B)	**7.** (C)
8. (D)	**9.** (C)	**10.** (D)	**11.** (B)	**12.** (D)	**13.** (D)	**14.** (B)
15. (C)	**16.** (B)	**17.** (A)	**18.** (C)	**19.** (A)	**20.** (D)	**21.** (B)
22. (C)	**23.** (D)	**24.** (C)	**25.** (A)	**26.** (D)	**27.** (C)	**28.** (B)
29. (C)	**30.** (A)	**31.** (B)	**32.** (C)	**33.** (B)	**34.** (D)	**35.** (A)
36. (C)	**37.** (C)	**38.** (D)	**39.** (B)	**40.** (C)	**41.** (B)	**42.** (A)
43. (A)	**44.** (B)					

EXERCISE - 2

1. (A)	**2.** (C)	**3.** (C)	**4.** (C)	**5.** (C)	**6.** (A)	**7.** (D)
8. (D)	**9.** (B)	**10.** (A)	**11.** (B)	**12.** (A)	**13.** (C)	**14.** (C)

EXERCISE - 3

1. (B)	**2.** (A)	**3.** (D)	**4.** (B)	**5.** (A)	**6.** (C)	**7.** (D)
8. (D)	**9.** (C)	**10.** (D)	**11.** (C)	**12.** (A)	**13.** (B)	**14.** (C)
15. (B)	**16.** (D)	**17.** (A)	**18.** (D)	**19.** (B)	**20.** (C)	**21.** (C)
22. (B)	**23.** (C)	**24.** (C)	**25.** (D)	**26.** (B)	**27.** (B)	**28.** (C)

Revision Plan

We emphasis that every student should prepare his/her own revision plan. For this purpose there is Revision Plan Section in each chapter which student should prepare while going thorugh the study material. This will be useful at the time of final revision before final exam for quick & effective revision.

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Exercise-1		
Exercise-2		
Exercise-3		
Exercise-4		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Step-3: Solve Questions of Column B

Step-4: Solve questions from other Question Bank, Problem book etc.

Online Solutions

Self explanatory and detailed soltuion of all excercises above are available on Career Point website **www.careerpoint.ac.in**

MOTION IN ONE DIMENSION

EXERCISE-1

Answer Key & Solution

Question Number	Solution	Question Number	Solution	Question Number	Solution	Question Number	Solution
1	Click Here	12	Click Here	23	Click Here	34	Click Here
2	Click Here	13	Click Here	24	Click Here	35	Click Here
3	Click Here	14	Click Here	25	Click Here	36	Click Here
4	Click Here	15	Click Here	26	Click Here	37	Click Here
5	Click Here	16	Click Here	27	Click Here	38	Click Here
6	Click Here	17	Click Here	28	Click Here	39	Click Here
7	Click Here	18	Click Here	29	Click Here	40	Click Here
8	Click Here	19	Click Here	30	Click Here	41	Click Here
9	Click Here	20	Click Here	31	Click Here	42	Click Here
10	Click Here	21	Click Here	32	Click Here	43	Click Here
11	Click Here	22	Click Here	33	Click Here	44	Click Here

GRAVITATION

JEE MAIN SYLLABUS

1. *Newton's Law Of Gravitation*
2. *Gravitational and Inertial Mass*
3. *Acceleration Due to Gravity*
4. *Gravitational Field And Gravitational Field Intensity*
5. *Gravitational Potential*
6. *Gravitational Potential Energy*
7. *Satellite*
8. *Geo-Stationary Satellites*
9. *Escape Velocity*
10. *Kepler's Laws*

Revision Plan
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EXERCISE	COLUMN A	COLUMN B
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Exercise-2		
Exercise-3		
Exercise-4		

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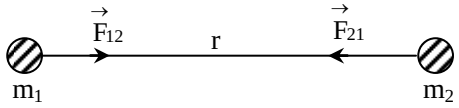
Gravitation

KEY CONCEPT

1. Newton's law of Gravitation

(a) Every two objects in the universe attract each other. This force of attraction is called *'Gravitational force'*.

(b) The force of attraction between any two material particles is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. It acts along the line joining the two particles.



$\therefore \quad F \propto m_1 m_2$

and $F \propto \frac{1}{r^2}$

or $F = \frac{G m_1 m_2}{r^2}$

(c) G is the constant of proportionality which is called 'Newton's gravitation constant.

$G = 6.67 \times 10^{-11}\ Nm^2 / kg^2$

$G = 6.67 \times 10^{-8}$ dyne cm^2 / gm^2

(d) Dimensional formula of G is $[M^{-1} L^3 T^{-2}]$

(e) In vector form-

The force exerted by point mass (2) on point mass (1) will be -

$$\vec{F}_{12} = \frac{G m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

direction of $\hat{r}_{12}$ is from 1 to 2.

$$\vec{F}_{21} = \frac{G m_1 m_2}{r_{12}^3} s . \vec{r}_{21},$$

similarly $\vec{F}_{12} = -\frac{G m_1 m_2}{r_{21}} \vec{r}_{21}$.

(f) $\vec{F}_{21} = -\vec{F}_{12}$

but, $|\vec{F}_{21}| = |\vec{F}_{12}|$

From above two expression we can conclude that the force exerted between two particles is equal in magnitude but opposite in direction.

(g) Gravitational force is the weakest force in nature.

Note :

The ratio of gravitational to electrostatic force between two electrons is of the order.

$$\frac{F_g}{F_e} = 10^{-43}.$$

(h) The range of this force is maximum upto infinity.

(i) It is due to very small value of G that we do not experience the gravitational force in our daily life. But masses of celestial bodies are so large that the magnitude of the force of attraction between them is appreciable. In the motion of planets and satellites, this force supplies the necessary centripetal force due to which earth revolves around the sun and moon around the earth.

(j) If the density of the earth is assumed to be uniform and a particle moves inside the earth then the gravitational force decreases because the shell of the material lying outside the particle's radial position would not exert any force on the particle.

2. Gravitational and Inertial mass

2.1 Inertial mass : When mass is defined on the property of inertia, it is termed as inertial mass.

2.2 Gravitational mass : When mass is defined on the property of gravity, it is called gravitational mass.

2.3 Properties of Inertial mass :

(i) It is equal to the ratio of magnitude of external force applied on the body to the acceleration produced in it by that force.

$m = F/a$.

(ii) It is proportional to the quantity of matter present in the body.

(iii) It is independent of shape, size and state of the body.

(iv) It is not affected by the presence of other bodies near it.

(v) When various masses are put together, the inertial masses add according to the scalar laws irrespective of the material of the bodies involved.

Point to remember :

- It is found that the ratio of two gravitational masses is same as the ratio of their inertial masses.
- If a ball is dropped from the hole passing though the two poles and the centre of earth then it will do S.H.M. in the tunnel.

3. Acceleration due to Gravity

(a) Acceleration produced in a body due to the force of gravity is termed as acceleration due to gravity.

(b) The acceleration due to gravity is the rate of increase of velocity of a body falling towards the earth.

(c) The acceleration due to gravity is equal to the force by which earth attracts a body of unit mass towards its centre.

(d) Let 'm' be the mass of body and 'F' be the force of attraction at a distance 'r' from the centre of earth, then acceleration due to gravity

(e) at that place will be.

$g = \frac{F}{m} = \frac{GM_e}{r^2}$, where M_e = mass of earth.

(f) The expression $g = \frac{GM_e}{r^2}$ is free from 'm' (mass of body). This means that the value of 'g' does not depend upon the shape, size and mass of the body. Hence if two bodies of different masses, shapes and sizes are allowed to fall freely, they will have the same acceleration. If they are allowed to fall form the same height, they will reach the earth simultaneously.

(g) The acceleration of a body on the surface of the earth is $g = 9.80\ m/s^2$ or $981\ cm/s^2$.

(h) Dimensional formula of g is $[M^0 L^1 T^{-2}]$.

(i) The value of acceleration due to gravity depends on the following factors.

(1) Height above the earth surface.

(2) Depth below the earth surface.

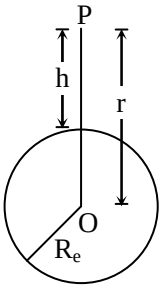
(3) Shape of the earth.

(4) Axial rotation of the earth.

(1) Height above the surface of earth :

(i) As we go above the surface of the earth, the value of 'g' decreases.

Consider a point P at a distance r form the centre of earth.



The acceleration due to gravity at point P is.

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$

$$\Rightarrow\ g' = g\left(\frac{R_e}{R_e+h}\right)^2 = \frac{gR_e^{\,2}}{r^2}$$

$\therefore\ g' < g$

(ii) As we go above the surface of the earth, the value of 'g' decreases. $g' \propto \frac{1}{r^2}$ for $r > R_e$

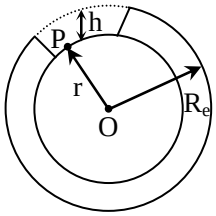
(iii) If $h << R_e$, (according to binomial expansion) $g' = g\left(1-\frac{2h}{R_e}\right)$

(iv) If $r = \infty$, $g' = 0$. At infinite distance form the earth, the value of 'g' becomes zero.

(v) Value of g at the surface of earth (h = 0)

$\Rightarrow g = \frac{GM_e}{R_e^2}$

(2) Below the surface of earth :



(i) The value of 'g' decreases on going below the surface of the earth.

(ii) The value of 'g' at a distance h below this earth's surface be g_h

and 'g' at the earth's surface then -

$g_h = g\left(1-\frac{h}{R_e}\right) = g\ (R_e - h)/R_e = \frac{gr}{R_e}$

i.e $g_h < g$

r is the distance form the centre of the earth ($r < R_e$), $r = R_e - h$.

(iii) If d is the density of the earth then the force on pt. P is

$g_h = \frac{M_r G}{r^2}$ where $M_r = \frac{4}{3}\pi (R_e - h)^3 d$;

$r^2 = (R_e - h)^2 \Rightarrow g_h = \frac{4}{3}\pi G (R_e - h)d$

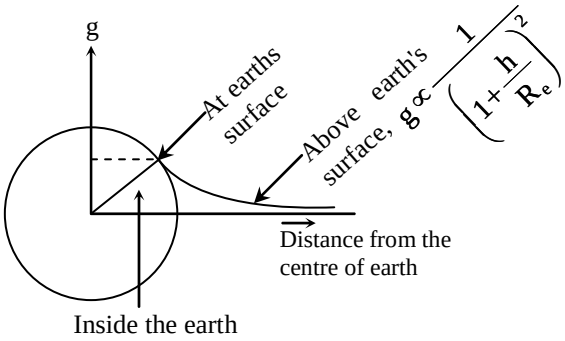
(iv) At the centre of the earth,

$h = R_e$ (i.e r = 0) so g = 0.

(v) Value of 'g' is maximum at the surface of earth.

(vi) Graphical representation of variation in the value of g

Variation of g with distance r from the centre of earth

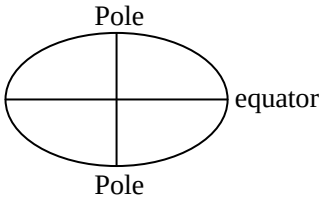


Variation in value of 'g' on the surface of earth :

It is due to two reasons :

(3) Due to shape of earth :

(i) The earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. Now, we know that $g \propto 1/R_e^2$, therefore the value of g at the equator is minimum and the value of g at the poles is maximum ($\because$ Radius at poles is < Radius at equator,)



$g_e = \frac{GM}{R_e^2}$

(At equator)

$g_p = \frac{GM}{R_p^2}$

(At pole)

$\because R_e > R_p$

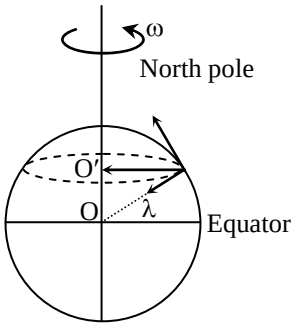
Hence, $g_e < g_p$

(ii) $\frac{g_p}{g_e} = \frac{R_e^2}{R_p^2}$

(iii) Numerical value of R_p is twenty one kilometre less than R_e. Therefore

$g_p - g_e = 0.02 m/s^2$.

(4) Due to rotation of earth :



(i) Earth is rotating about its own axis form west to earth with an angular velocity ω.

(ii) On a latitude λ, point P is moving in circle with radius 'r'. If we keep a body then some part of its gravitational force will be used up for providing centripetal force, therefore there is reduction in total gravitational force. As a result of this, value of 'g' decreases.

(iii) If ω is the angular velocity of rotation of the earth, R_e is radius of the earth, then the observed value of g at the latitude is represented by g' then.

$$g' = g_o - \omega^2 R_e \cos^2 \lambda$$

or $g' = g_o - 0.0337 \cos^2 \lambda$

where g_0 is value of 'g' at the poles.

(iv) At equator, $\lambda = 0^o$

$g' = g_0 - \omega^2 R_e$ (Minimum value)

$= g - 0.0337$

(v) At poles, $\lambda = 90°$

$\therefore \cos = 0$

$g' = g$ (Maximum value)

(vi) From the above expressions we can conclude that the value of 'g' at the surface of earth is maximum at poles and minimum at the equator. Therefore weight of bodies is maximum at the poles and will go on decreasing towards the equator. (it is minimum at the equator).

(vii) If earth stops rotating about its axis ($\omega = 0$), the value of g will increase everywhere, except at the poles. On the contrary, if there is increase in the angular velocity of earth, then except at the poles the value of 'g' will decrease at all places.

(viii) Maximum effect of rotation takes place at the equator while at poles, there is no effect.

(ix) If $\omega = \sqrt{\frac{g}{R_e}}$ then, at equator weight of body will become zero but its mass remains unaltered.

- That means if the earth starts rotating with an angular speed 17 times the present,
- If $g_{equator} = 0$, in this condition , time period of earth's rotation will become 1.41 hours instead of 24 hours.

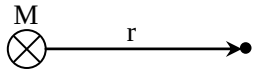
4. Gravitational field and Gravitational field intensity

(a) It is defined as the space around the attracting body, in which its attraction (gravitational) can be experienced.

(b) ***Intensity of gravitational field or gravitational field strength*** - It is defined as the force experienced by unit mass place at any point in the gravitational field.

(c) Gravitational field is a vector quantity.

(d) Suppose a body of mass M is placed at a distance r, then intensity of gravitational field



at point P will be $\vec{E} = \frac{GM_e}{r^2}(-\hat{r})$

(e) Unit of gravitational field strength is Newton/kg or m/sec^2.

Dimension formula is $[M^0L^1T^{-2}]$.

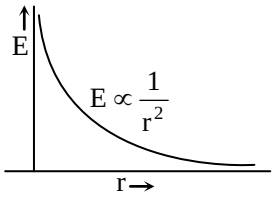
(f) As the distance (r) increases, gravitational field strength decreases.

At r = ∞, value of intensity of gravitational field becomes zero.

(g) Intensity of gravitational field at a distance r form the centre of earth is

$$E = \frac{GM_e}{r^2} = g.I$$

NOTE : From this expression



$$E = \frac{GM_e}{r^2} = g$$

it is clear that the intensity of gravitational field at any place is equal to acceleration due to gravity.

(h) Change of intensity of gravitational field due to a point mass with respect to distance -

$$E = \frac{GM_e}{r^2}$$

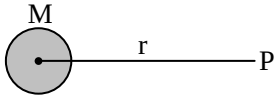
(i) Relation between gravitational field and gravitational potential -

$$E = -\Delta V \qquad \Rightarrow E = -\frac{dV}{dr}$$

5. Gravitational Potential

(a) The work done in bringing a unit mass form infinity to a point in the gravitational field is called the 'gravitational potential' at that point.

(b) Gravitational potential at a point P distance r form a point mass 'M' will be $V = -\frac{GM}{r}$



(c) Unit of Gravitational potential is Joule/kg

(d) Dimensional formula of Gravitational potential is $[M^0 L^2 T^{-2}]$.

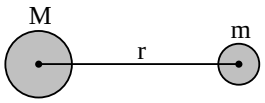
(e) Gravitational potential is a scalar quantity.

(f) At r = ∞, V = 0.

6. Gravitational Potential Energy

(a) The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the field.

(b) The gravitational potential energy of mass 'm' in the gravitational field of mass M at a distance r form it is



$$U = -\frac{GMm}{r}$$

where r is distance between M and m.

(c) At any place in gravitational field, gravitational potential is V, then the gravitational potential energy of a mass 'm' at that place will be -

$$U = -mV.$$

(d) The gravitational potential energy of a particle of mass 'm' at a point distant 'r' from the centre of earth is

$$U = -\frac{GM_em}{r}, \text{ if } r > R_e$$

$$= -\frac{GM_em(3R_e^2 - r^2)}{2R_e^3}, \text{ if } r < R_e$$

(e) Force between two particle if their potential energy is U, is $F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}\right)$

$= -\frac{GMm}{r^2}$ minus sign. indicates that the force on the bodies is towards each other.

Note :- If a particle is at a height h from earth's surface and R_e be the radius of earth,

$r = R_e + h$

$$U = -\frac{GM_em}{R_e + h}$$

(f) It is a scalar quantity and its value is always negative.

(g) It's unit is Joule or Erg.

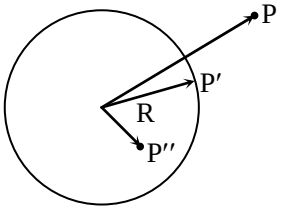
(h) Gravitational potential energy of a mass at infinite distance from earth is zero, and at all the other points it is less than zero, i.e. it is negative.

6.1 Intensity of Gravitational field and gravitational potential due to hollow sphere

6.1.1 Hollow Sphere :

(i) Let OP = r

If point P is situated outside the hollow sphere, then OP = r > R



- $E_{out} = -\frac{GM}{r^2}$
- $V_{out} = -\frac{GM}{r}$

(ii) If point P is situated on the surface of sphere, then OP' = r = R

- $E_{surface} = -\frac{GM}{R^2}$
- $V_{surface} = -\frac{GM}{R}$

(iii) If point P is inside the hollow sphere, then

OP" = r < R

- $E_{in} = 0$
- $V_{in} = -\frac{GM}{R}$

Note : Gravitational field intensity inside a hollow sphere is zero but gravitational potential is constant and is equal to the potential at the surface.

6.1.2 Solid Sphere :

Let OP = r

(i) If point P is situated outside the sphere, then OP = r > R

- $E_{out} = -\frac{GM}{r^2}$
- $V_{out} = -\frac{GM}{r}$

(ii) If point P is situated on the surface of sphere, then OP = r = R

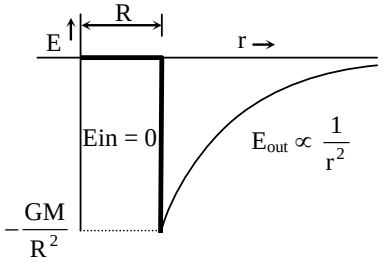
- $E_{surface} = -\frac{GM}{R^2}$
- $V_{surface} = -\frac{GM}{R}$

(iii) If point P is situated inside the sphere, then OP = r < R_e

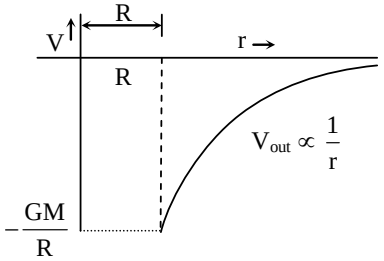
- $E_{in} = -\frac{GMr}{R^3}$
- $V_{in} = -\frac{GM(3R^2 - r^2)}{2R^3}$

Note : $V_{centre} = 1.5\ V_{surface}$

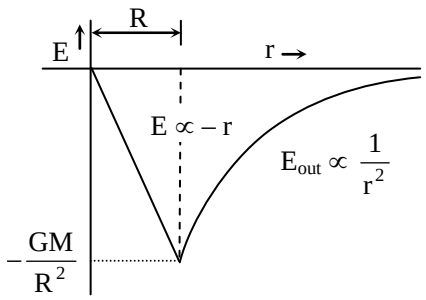
Graphical Representation of Gravitational Field Intensity For Hollow Sphere



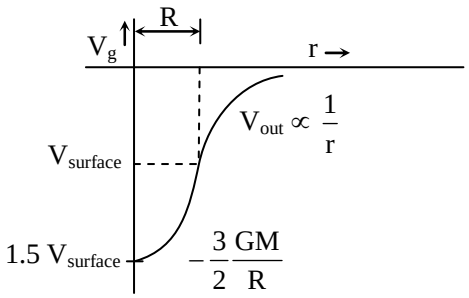
Graphical Representation of Gravitational potential For Hollow Sphere



For Solid Sphere



For Solid Sphere



7. Satellite

(a) Celestial bodies revolving round the gravitational field of the planets is called satellite.

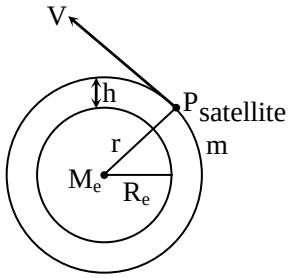
(b) Satellites are of two types -

(i) Natural satellites - As moon is a satellite of the earth.

(ii) Artificial satellites - They are launched by man such as Rohini, Aryabhatt etc.

(c) Let a satellite of mass 'm' revolves in a circular orbit with radius 'r' around the earth.

The necessary centripetal force needed for circular motion is provided by the gravitational force of the earth.



so, $\frac{GM_em}{r^2} = \frac{mv^2}{r}$

Where M_e = mass of earth

v = Orbital velocity of satellite

r = Radius of satellite's orbit = $R_e + h$ = orbital radius

R_e = Radius of earth

h = The height of the satellite above the earth's surface

g = Acceleration due to gravity on the surface of the earth.

(d) **Orbital velocity of satellite :**

(i) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM_e}{R_e + h}} = \sqrt{\frac{gR_e^2}{R_e + h}}$

(ii) From this expression, $\sqrt{\frac{gR_e^2}{R_e + h}}$, it is clear that orbital velocity does not depend on the mass of satellite but it depends on the height of the satellite above the earth's surface (h). Greater the height of satellite, smaller is the orbital velocity.

Note : This proves that keplers II Law is true.

(iii) If a satellite is very close to the earths surface (h << Re), then h will be negligible as compared to R than the orbital speed of satellite is given by

$$v = \sqrt{\frac{GM_e}{R}} = \sqrt{gR}$$

= 7.92 km/sec. (≅ 8km/sec.).

(e) Period of Revolution :

(i) The time taken by the satellite for completing one revolution of earth is called as period of revolution of satellite.

(ii) Period of Revolution of a satellite is

$$T = \frac{2\pi r}{v} = \frac{2\pi(R_e + h)}{v},$$

where 'T' is the time period of a satellite at a height 'h'.

(iii) $T = 2\pi\sqrt{\frac{r^3}{GM_e}} = 2\pi\sqrt{\frac{r^3}{gR_e^2}}$

$$= 2\pi\sqrt{\frac{(h+R_e)^3}{gR_e^2}} = 2\pi\sqrt{\frac{R_e}{g}}\left(1+\frac{h}{R_e}\right)^{3/2}$$

(iv) It is evident from the above expression that $T^2 \propto r^3$ i.e kepler's III law is true for circular motion also.

(v) For a satellite revolving very close to the surface of earth ($h << R_e$),

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.4 \text{ min.}$$

therefore, the minimum time period of the satellite revolving very close to the surface of earth is 84.4 min.

(vi) From the expression $'T = 2\pi\sqrt{\frac{r^3}{GM_e}}$ we can say that time period of satellite depends on its orbital radius $T^2 \propto r^3$ As the radius increases, simultaneously time period also increase.

(f) Energy of Satellite : When satellite is revolving in the orbit of radius 'r' then

(i) Potential energy of satellite :

$$P.E = -\frac{GM_em}{r}, \text{ where } r = h$$

(ii) Kinetic energy of satellite :

$$K.E = \frac{1}{2}mv^2 = \frac{GM_em}{2r}$$

(iii) Total energy of satellite

$$= K.E + P.E = \frac{-GM_em}{2r}$$

(iv) We can say that : Total energy of satellite $= \frac{1}{2}$ (Potential energy of satellite)

= – Kinetic Energy of satellite.

(g) Binding Energy of satellite :

(i) Binding energy is the energy given to satellite in order that the satellite escape away form its orbit.

Binding Energy =

$$-\text{ Total Energy} = \frac{GM_em}{2r}$$

(i.e equal to kinetic energy)

If energy equals to $\frac{GM_em}{2r}$, is provided to the satellite, it will escape away from the gravitational field of the planet.

(ii) Unless a revolving satellite gets extra energy, it would not leave its orbit. If the kinetic energy of a satellite happens to increase to two times, the satellite would escape.

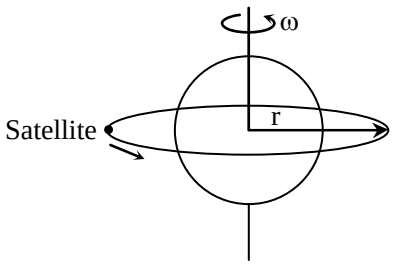
(iii) If the orbital velocity of a satellite revolving close to the earth happens to increase to $\sqrt{2}$ times, the satellite would escape. That means if orbital velocity increases to 41.4%, satellite would leave the orbit.

(iv) Total energy of satellite is always negative. When the energy of the satellite is negative, it moves in either a circular or an elliptical orbit.

(v) Binding Energy = Kinetic Energy =

$$-\text{ (Total Energy)} = -\frac{(\text{PotentialEnergy})}{2}$$

8. Geo-Stationary Satellites



(a) Such satellites which are stationary with respect to an observer on earth are termed as Geostationary satellites. They are also called Parking satellites.

(b) The direction of rotation of geo-stationary satellites is from west to east, the time period is 24 hours and its angular velocity is same as that of axial velocity of earth, revolving around its axis.

(c) Geo-stationary satellites can be launched just above the equator.

(d) The radius of orbit of Geo-stationary satellite is r = 42,000 km and its height above the surface of earth is h = 36, 000 km.

(e) Different values of satellite -

- Angular Velocity (ω) = 7.1 x 10^{-5} rad/sec
- Linear Velocity (v) = 3.1 km/sec.
- Time Period (T) = 24 hours.
- Height above the earth's surface (h) = 36,000km (approx.)

(f) At time t, the angular displacement of earth and Geo stationary satellite is same.

(g) Angular momentum of satellite is conserved and it is equal to

$$J = mvr = mr\sqrt{\frac{gR_e^{\,2}}{r}} = mR_e\sqrt{gr} = m\sqrt{GM_e r}$$

(h) Satellites behaves like freely falling bodies towards planet.

(i) The satellite revolves around the earth in an orbit with earth as centre or a focus.

(j) If a packet is released form the satellite, it will not fall on the earth but will remain revolving in the same orbit with the same speed as the satellite.

(k) No gravitational force of satellite is used up for providing necessary centripetal force. Due to gravitational force the effective value of acceleration due to gravity becomes $g_{eff} = 0$, as a result effective weight becomes $w_{eff} = 0$, so the man sitting in the satellite enjoys weightlessens. Man experiences this weightlessness condition only when weight of satellite is very less therefore gravitational effect of satellite is negligible.

Although moon is also a satellite of the earth, but a person on moon does not feel weightlessness. Thereason is that the moon has a large mass and exerts a gravitational force on the person (and this is the weight of the person on the moon). On the other hand, the artificial satellite having a smaller mass does not exert gravitational force on the space-man.

Relation between velocity of Projection and shape of orbit

Shape of the satellite's orbit depends on its velocity.

$V_0 = \sqrt{\frac{GM_e}{R_e + h}}$; orbital velocity of the satellite's

Cases :

(i) **If $V < V_0$;** In this case satellite will leave its circular orbit and finally fall to earth following spiral path.

(ii) **If $V = V_0$;** In this case satellite will rotate in circular path.

(iii) **If $V_0 < V < \sqrt{2}\,V_0$;** In this case satellite will revolve around the earth in elliptical orbit.

(iv) **If $V = \sqrt{2}\,V_0$;** In this case satellite will leave the gravitational field of earth and escape away following a parabolic path.

(v) **If $V > \sqrt{2}\,V_0$;** In this case the satellite will escape, following a hyperbolical path.

9. Escape Velocity

(a) Escape velocity is the minimum velocity that should be given to the body to enable it to escape away form the gravitational field of earth.

(b) The energy given to the body to project it with the escape velocity is called the 'Escape Energy' or 'Binding Energy'.

(c) Total energy of a body is reduced to zero to enable it to escape away from the gravitational field of earth.

(d) The gravitational potential energy of a particle at the surface of earth. $= \frac{-GM_em}{R_e}$

Escape Energy or Binding Energy

$= + \frac{GM_em}{R_e}$ if thrown with the velocity v_e,

then $\frac{1}{2}mv_e^2 = \frac{GM_em}{R}$

Escape velocity for earth

$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2$ km/sec.

(e) The value of escape velocity does not depend upon the mass of the projected body, instead it depends on the mass and radius of the planet from which it is being projected.

(f) There is no atmosphere on the planets where the root-mean square velocities more than the escape velocity.

(g) The value of escape velocity does not depend on the angle and direction of projection instead it depends on density, mass and acceleration due to gravity of the planet.

10. Kepler's Laws

Kepler found important regularities in the motion of the planets. These regularities are known as Kepler's three laws of planetary motion.

10.1 First Law :

Every planet move around the sun in an elliptical orbit with sun at one of the focus. This is the law of orbits.

10.2 Second Law :

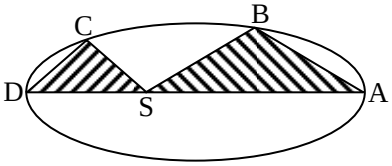
The line joining the sun to the planet sweeps out equal area in equal interval of time i,e. the area velocity is constant. This is the law of area. The second law tells us that the planet will move most slowly when it is farthest from sun and most rapidly when it is nearest to sun. So we can say that this law is similar to law of conservation of angular momentum.

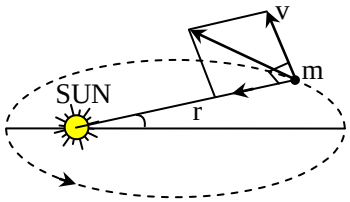
Where L = angular momentum

m = mass of planet

v = Linear velocity component $\perp$ to r

r = Distance of sun & planet.





Areal velocity $= \frac{L}{2m} = \frac{vr}{2}$

Conclusions from the figure :

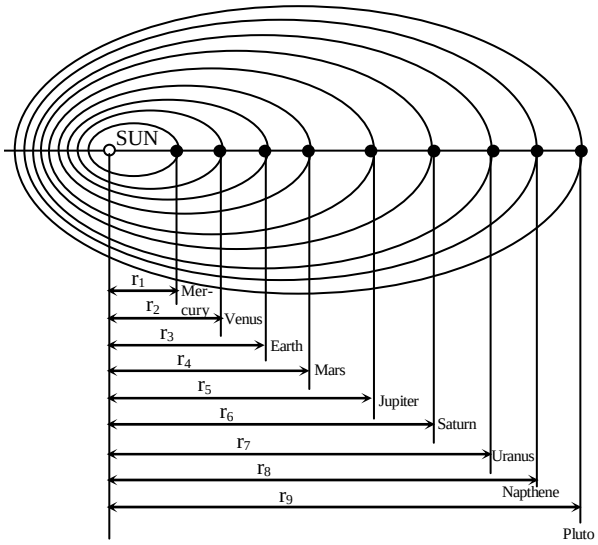
If a planet moves from A to B in time t_1, and from C to D in time t_2, then if

$t_1 = t_2$, Area of SBA = Area of SCD.

$t_1 > t_2$, Area of SBA > Area of SCD.

$t_1 < t_2$, Area of SBA < Area of SCD.

NOTE : When the planet is nearest to the sun, its speed is maximum and when it is farthest form the sun, then its speed is minimum.



10.3 Third Law :

The square of the period of revolution (T) of any planet around the sun is directly proportional to the cube of its average distance (r) from the sun. This is the law of periods.

$T^2 \propto r^3$

or $T^2 = kr^3$,

where k = constant

Thus, it is clear from the law that larger the distance of a planet from the sun, larger will be its period of revolution around the sun.

NOTE :

- Mercury is the nearest planet to the sun and its time period is 88 days while Pluto is the farthest planet and its time period is 248 years.
- Kepler's laws are valid for motion of satellites.
- For all the planets of the sun.

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \frac{T_3^2}{r_3^3} = \text{--...--} = \text{constant}$$

- When a planet comes close to the sun its linear velocity increases.

Important Points

- Relation between mass of earth and its density :

$$g = \frac{GM_e}{R_e^2} = \frac{G\frac{4}{3}\pi R_e^3 d}{R_e^2}$$

$$\therefore \quad d = \frac{3g}{4\pi G R_e} = 5.47 \text{ gm /cm}^3$$

d = density

- Mass of Earth (M_e) :

$$M_e = \frac{gR_e^2}{G} = 6.6 \times 10^{24} \text{kg (approx.)}$$

- Value of 'g' on other planets :

$$\frac{g}{g_e} = \left(\frac{M}{M_e}\right)\left(\frac{R_e}{R}\right)^2$$

- Comparison of masses of two planets :

$$\frac{M_1}{M_2} = \left(\frac{T_1}{T_2}\right)^2 \left(\frac{r_1}{r_2}\right)^3$$

where T_1 = Time period of Ist planet.

T_2 = Time period of IInd planet.

r_1 = Orbital radius of Ist planet.

r_2 = Orbital radius of IInd planet.

- Mass of Sun :

$$M_S = \frac{4\pi^2 r^3}{T^2 G} = 19.72 \times 10^{29} \text{ kg}$$

where T = Time period of Earth = 365 days.

r = Distance of earth form sun.

- If we throw a body from earth towards moon, then as body moves away from the surface of earth the value g_{earth} decreases, as a result the weight (W = mg on earth) decreases. At a

particular height $g_{earth} = g_{moon}$ therefore W = 0. Now above this height g_{moon} becomes effective and weight (W) increases till body reaches, at the surface of the moon.

- If two mass m and M moves towards each other due to gravitational force, from rest, then relative velocity of approach will be, $\sqrt{\frac{2G(M+m)}{r}}$ where r is separation between masses.
- Maximum height attained by a projectile (h) is given by : $h = \frac{v^2R}{2gR - v^2}$
- If h_1 is the maximum height attained by a man on a planet whose acceleration due to gravity is g_1 and h_2 is the maximum height attained on a planet whose acceleration due to gravity is g_2 then.

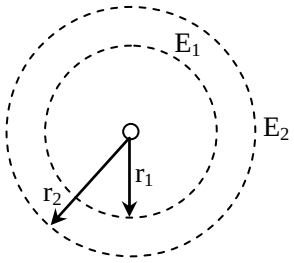
$$g_1h_1 = g_2 h_2$$

- On clock based on spring, the effect of acceleration due to gravity is negligible but on pendulum based clock effect of 'g' is noticed if 'g' increases, time period decreases and the clock moves fast if 'g' decreases time period increases and the clock moves slow.
- Work done in changing the orbit of satellite :

W = Change in total energy

$$\text{Total } E = -\frac{GM_em}{2r}$$

$$W = E_2 - E_1 = \frac{GM_em}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$



- If satellite is moved to higher orbit then potential energy, angular momentum, increases but kinetic energy, binding energy, orbital velocity, decrease.

SOLVED EXAMPLES

Ex.1 What will be the acceleration due to gravity on the surface of the moon if its radius is 1/4 th the radius of the earth and its mass is 1/80 th the mass of the earth.

(A) g/6 (B) g/5

(C) g/7 (D) g/8

Sol **(B)**

The acceleration due to gravity on the surface of the earth, in terms of mass M_e and radius R_e of earth, is given by

$$g = \frac{GM_e}{R_e^2}$$

if M_m be the mass of the moon, R_m its radius, then the acceleration due to gravity on the surface of the moon will be given by

$$g' = \frac{GM_m}{R_m^2}$$

Dividing eq. (ii) by eq. (i), we get

$$\frac{g'}{g} = \frac{M_m}{M_e}\left(\frac{R_e}{R_m}\right)^2 = \frac{1}{80} \times \left(\frac{4}{1}\right)^2 = \frac{1}{5}$$

$g' = g/5$.

Ex.2 At what height above the earth's surface the acceleration due to gravity will be 1/9 th of its value at the earth's surface ? Radius of earth is 6400 km.

(A) 12800km (B) 1280km

(C) 128000km (D) 128km

Sol **(A)**

If g be the acceleration due to gravity at the surface of the earth, then its value at a height h above the earth's surface will be -

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$

Here $\frac{g'}{g} = \frac{1}{9}$ $\therefore$ $\frac{1}{9} = \frac{1}{\left(1+\frac{h}{R_e}\right)^2}$

or $1 + \frac{h}{R_e} = 3$

or $h = 2R_e = 2 \times 6400 = 12800$ km.

Ex.3 Four particles, each of mass m, are placed at the corners of square and moving along a circle of radius r under the influence of mutual gravitational attraction. The speed of each particle will be -

(A) $\sqrt{\frac{Gm}{r}}\ (2\sqrt{2}+1)$

(B) $\sqrt{\frac{Gm}{r}}$

(C) $\sqrt{\frac{Gm}{r}\left(\frac{2\sqrt{2}+1}{4}\right)}$

(D) $\sqrt{\frac{2\sqrt{2}Gm}{r}}$

Sol **(C)**

Resultant force on particle '1'

$F_r = \sqrt{2}\ F + F'$

or $F_r = \sqrt{2}\ \frac{Gm^2}{2r^2} + \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$

or $v = \sqrt{\frac{Gm}{r}\left(\frac{2\sqrt{2}+1}{4}\right)}$

Ex.4 Three particles of equal mass m are situated at the vertices of an equilateral triangle of side ℓ. What should be the velocity of each particle, so that they move on a circular path without changing ℓ -

(A) $\sqrt{\frac{GM}{2\ell}}$ (B) $\sqrt{\frac{GM}{\ell}}$

(C) $\sqrt{\frac{2GM}{\ell}}$ (D) $\sqrt{\frac{GM}{3\ell}}$

Sol **(B)**

The resultant gravitational force on each particle provides it the necessary centripetal force

$\therefore$ $\frac{mv^2}{r} = \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ} = \sqrt{3}\ F,$

But. $r = \frac{\sqrt{3}}{2}\ \ell \times \frac{2}{3} = \frac{\ell}{\sqrt{3}}$,

$\therefore$ $v = \sqrt{\frac{GM}{\ell}}$

Ex.5 If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would -

(A) decrease,

(B) remain unchanged,

(C) increase.

(D) None of these

Sol **(C)**

Consider the case of a body of mass m placed on the earth's surface (mass of the earth M and radius R). If g is acceleration due to gravity, then

$$mg = G\frac{M_e m}{R^2} \quad \text{or} \quad g = \frac{GM_e}{R^2}$$

where G is universal constant of gravitation.

Now when the radius is reduced by 1%, i.e., radius becomes 0.99 R, let acceleration due to gravity be g', then

$$g' = \frac{GM_e}{(0.99R)^2}$$

From equation (A) and (B), we get

$$\frac{g'}{g} = \frac{R^2}{(0.99R)^2} = \frac{1}{(0.99)^2}$$

$$g' = g \times \left(\frac{1}{0.99}\right)^2 \text{ or } g' > g$$

Thus , the value of g is increased.

Ex.6 At what height above the Earth's surface does the force of gravity decrease by 10%. Assume radius of earth to be 6370 km.

(A) 350km. (B) 250km.

(C) 150km. (D) 300km.

Sol **(A)**

Force of gravity at surface of earth,

$F_1 = Gm\ M/R^2$ ....(1)

Force of gravity at height H is

$F_2 = Gm\ M\ (R + H)^2$ ....(2)

Dividing (A) by (B) and Rearranging

$$H = R\left(\sqrt{\frac{F_1}{F_2}} - 1\right) = 350 \text{ km where } (F_2 = .9F_1)$$

Ex.7 The speed with which the earth would have to rotate on its axis so that a person on the equator would weight 3/5 th as much as at present, will be (Take the equatorial radius as 6400 km.)

(A) 7.8×10^{-3} radian/sec.

(B) 7.8×10^{-4} radian/sec.

(C) 7.8×10^{-5} radian/sec.

(D) 7.8×10^{-2} radian/sec.

Sol **(B)**

At present the weight of the person on the equator is nearly the same which would have been if the earth were stationary. Suppose, for the weight to remain 3/5 th, the angular speed of earth is ω. Then according to the formula

$g' = g - R_e\omega^2$, we have

$$\frac{3}{5}mg = mg - m\,R_e\omega^2.$$

$$\therefore\ \omega = \sqrt{\left(\frac{2g}{5R_e}\right)} = \sqrt{\left(\frac{2\times 9.8 m/s^2}{5\times(6400\times 10^3 m)}\right)}$$

$= 7.8 \times 10^{-4}$ radian/sec.

Ex.8 With what velocity must a body be thrown upward form the surface of the earth so that it reaches a height of 10 R_e? Earth's mass $M_e = 6 \times 10^{24}$ kg, radius $R_e = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ N-m^2/kg^2.

(A) 10.7×10^4 m/s (B) 10.7×10^3 m/s

(C) 10.7×10^5 m/s (D) 1.07×10^4 m/s

Sol **(D)**

Let m be the mass of the body. The gravitational potential energy of the body at the surface of the earth is

$$U = -\frac{GM_e m}{R_e}$$

The potential energy at a height 10 R_e above the surface of the earth will be

$$U' = -\frac{GM_e m}{(R_e + 10R_e)}$$

increase in potential energy

$$U' - U = -\frac{GM_e m}{11R_e} + \left(\frac{GM_e m}{R_e}\right)$$

$$= \frac{10}{11}\frac{GM_e m}{R_e}$$

This increase will be obtained from the initial kinetic energy given to the body. Hence if the body be thrown with a velocity then

$$\frac{1}{2}m\upsilon^2 = \frac{10}{11}\frac{GM_e m}{R_e}$$

$$\Rightarrow \quad \upsilon = \sqrt{\frac{20Gm_e}{11R_e}}$$

Substituting the given values, we get

$$\upsilon = \sqrt{\left(\frac{20\times(6.67\times10^{-11})\times(6\times10^{24})}{11\times(6.4\times10^{6})}\right)}$$

$$= 1.07 \times 10^4 \text{ m/s.}$$

Ex.9 The radius of the earth is R_e and the acceleration due to gravity at its surface is g. The work required in raising a body of mass m to a height h form the surface of the earth will be -

(A) $\dfrac{mgh}{\left(1-\dfrac{h}{R_e}\right)}$ (B) $\dfrac{mgh}{\left(1+\dfrac{h}{R_e}\right)^2}$

(C) $\dfrac{mgh}{\left(1+\dfrac{h}{R_e}\right)}$ (D) $\dfrac{mg}{\left(1+\dfrac{h}{R_e}\right)}$

Sol (C)

Let M_e be the mass of the earth. The work required

$$W = GM_e\, m\left[\frac{1}{R_e}-\frac{1}{R_e+h}\right]$$

$$= \frac{GM_e mh}{R_e(R_e+h)} = \frac{gR_e^2 mh}{R_e(R_e+h)}$$

$$[\because\ GM_e = gR_e^2] = \frac{mgh}{\left(1+\dfrac{h}{R_e}\right)}$$

Ex.10 If the satellite is stopped suddenly in its orbit which is at a distance = radius of earth from earth's surface and allowed to fall freely into the earth, the speed with which it hits the surface of earth will be -

(A) 7.919 m/sec (B) 7.919 km/sec

(C) 11.2 m/sec (D) 11.2 km/sec

Sol **(B)**

From conservation of energy,

The energy at height h = Total energy at earth's surface

$$0 - \frac{GMm}{R+h} = \frac{1}{2}mv^2 - \frac{GMm}{R},$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{R^2 g}{R}} = \sqrt{Rg}$$

$$= \sqrt{6400\times10^3\times9.8} = 7.919 \times 10^3 \text{ m/s}$$

$$= 7.919 \text{ km/sec .}$$

Ex.11 A projectile is fired vertically upward from the surface of earth with a velocity Kv_e, where v_e is the escape velocity and K < 1. Neglecting air resistance, the maximum height to which it will rise measured from the centre of the earth is - (where are R = radius of earth)

(A) $\dfrac{R}{1-K^2}$ (B) $\dfrac{R}{K^2}$

(C) $\dfrac{1-K^2}{R}$ (D) $\dfrac{K^2}{R}$

Sol **(A)**

If a body is projected from the surface of earth with a velocity v and reaches a height h, then using law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{mgh}{1+h/R}.$$

Given $v = Kv_e = K\sqrt{2gR}$ and $h = r - R$

Hence, $\dfrac{1}{2}mK^2(2gR) = \dfrac{mg(r-R)}{1+\dfrac{r-R}{R}}$

or $\quad r = \dfrac{R}{1-K^2}$

Ex.12 A body of mass 100 kg falls on the earth from infinity. What will be its energy on reaching the earth ? Radius of the earth is 6400 km and g = 9.8 m/s^2. Air friction is negligible.

(A) 6.27×10^9 J (B) 6.27×10^{10} J

(C) 6.27×10^{10} J (D) 6.27×10^7 J

Sol **(A)**

A body projected up with the escape velocity will go to infinity. Therefore, the velocity of the body falling on the earth from infinity will be. Now, the escape velocity on the earth is -

$$\upsilon_e = \sqrt{gR_e} = \sqrt{2\times(9.8 m/s^2)\times(6400\times10^3 m)}$$

$= 1.2 \times 10\ 10^4$ m/s = 11.2 km/s.

The kinetic energy acquired by the body is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\times 100\ kg \times (11.2\times10^3\ m/s)^2$$

$= 6.27 \times 10^9$ J.

Ex.13 A satellite is revolving in an orbit close to the earth's surface. Taking the radius of the earth as 6.4×10^6 meter, the value of the orbital speed and the period of revolution of the satellite will respectively be -

(g = 9.8 meter/sec^2)

(A) 7.2 km/sec., 84.6 minutes

(B) 2.7 km/sec., 8.6 minutes

(C) .72 km/sec., 84.6 minutes

(D) 7.2 km/sec., 8.6 minutes

Sol **(A)**

Orbital speed,

$$\upsilon_o = \sqrt{g R_e} = \sqrt{9.8\times(6.4\times10^6)}$$

$= 7.2 \times 10^3$ m/s = **7.2 km/s**.

Period of revolution, $T = 2\pi\sqrt{R/g}$

$= 2 \times 3.14 \cdot \sqrt{6.4\times10^6/9.8} = 5075$ s

= 84.6 minutes.

Ex.14 If the period of revolution of an artificial satellite just above the earth be T and the density of earth be ρ, then

(A) ρT^2 is a universal constant.

(B) ρT^2 varies with time

(C) $\rho T^2 = \frac{3\pi}{G}$

(D) $\rho T^2 = 3\pi \times G$

Sol **(A), (C)**

If the period of revolution of a satellite about the earth be T, then

$$T^2 = \frac{4\pi^2(R_e + h)^3}{GM_e}$$

where h is the height of the satellite from earth's surface.

$$M_e = \frac{4\pi^2(R_e + h)^3}{GT^2}$$

The satellite is revolving just above the earth, hence h is negligible compared to R_e.

$$M_e = \frac{4\pi^2 R_e^3}{GT^2}$$

But $M_e = \frac{4}{3}\pi R_e^3 \rho$ where ρ is the density of the earth. Thus

$$\frac{4}{3}\pi R_e^3 \rho = \frac{4\pi^2 R_e^3}{GT^2}$$

$$\therefore \quad \rho T^2 = \frac{3\pi}{G}.$$

which is universal constant. To determine its value,

$$\rho T^2 = \frac{3\pi}{G} = \frac{3\times3.14}{6.67\times10^{-11}\ m^3/kg-s^2}$$

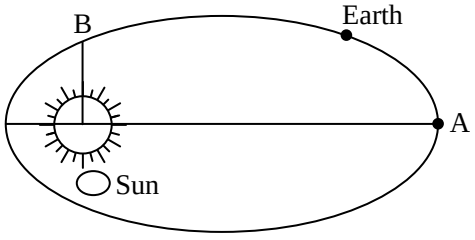
Ex.15 The mass of the sun if the mean radius of earth's orbit is 1.5×10^8 km and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 will be-

(A) 2×10^{10} kg (B) 3×10^{10} kg

(C) 2×10^{30} kg (D) 3×10^{30} kg

Sol **(C)**

We know that, $v = \sqrt{\frac{GM}{r}}$,



$$\therefore \quad T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$$

or $M = \frac{4\pi^2 r^3}{GT^2}$

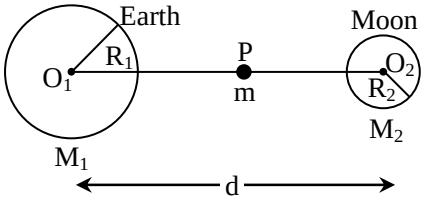
(As T = 1 year = 3.15×10^7 sec)

Ex.16 The masses and the radius of the earth and the moon are M_1, M_2 and R_1, R_2 respectively their centres are at distance d apart. The minimum speed with which a particle of mass m should be projected form a point midway between the two centres so as to escape to infinity will be -

(A) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ (B) $\sqrt{\frac{G}{d}(M_1+M_2)}$

(C) $\sqrt{\frac{G}{2d}(M_1+M_2)}$ (D) $2\sqrt{\frac{G}{d}\frac{M_1}{M_2}}$

Sol (A) The P.E of the mass at d/2 due to the earth and moon is



$$U = -2\frac{GM_1m}{d} - 2\frac{GM_2m}{d}$$

or $U = -\frac{2Gm}{d}(M_1 + M_2)$ (Numerically)

$$\frac{1}{2} m V_e^2 = U$$

$$\Rightarrow V_e = 2\sqrt{\frac{G}{d}(M_1+M_2)}$$

Ex.17 The Jupiter's period of revolution round the sun is 12 times that of the earth. Assuming the planetary orbits to circular, how many times the distance between the jupiter and sun exceeds that between the earth and the sun.

(A) 5.242 (B) 4.242

(C) 3.242 (D) 2.242

Sol **(A)**

We know that $T^2 \propto a^3$

Given that $(12\ T)^2 \propto a_1^3$ and $T^2 \propto a_2^3$

$$\frac{a_1^3}{a_2^3} = \frac{(12T)^2}{T^2} = 144$$

or $\frac{a_1}{a_2} = (144)^{1/3} = 5.242$

Hence the jupiter's distance is 5.242 times that of the earth from the sun.]

Ex.18 The mean distance of mars from sun is 1.524 times the distance of the earth from sun. The period of revolution of mars around sun will be -

(A) 2.88 earth year (B) 1.88 earth year

(C) 3.88 earth year (D) 4.88 earth year

Sol **(B)**

We know that $T^2 \propto a^3 \Rightarrow T \propto (a)^{3/2}$

$$\frac{T_{mars}}{T_{earth}} = \left(\frac{a_{mars}}{a_{earth}}\right)^{\frac{3}{2}} = (1.524)^{3/2} = 1.88$$

As the earth revolves round the sun in one year and hence,

$T_{earth} = 1$ year.

$T_{mars} = T_{earth} \times 1.88 = 1 \times 1.88$

$= 1.88$ earth-year.

Ex.19 The semi-major axes of the orbits of Mercury and Mars are respectively 0.387 and 1.524 in astronomical units. If the period of Mercury is 0.241 year, what is the period of Mars.

(A) 1.2 years (B) 3.2 years

(C) 3.9 years (D) 1.9 years

Sol **(D)**

$$\frac{T_{mercury}}{T_{mars}} = \left(\frac{a_{mercury}}{a_{mars}}\right)^{3/2} = \left(\frac{0.387}{1.524}\right)^{3/2}$$

$$\therefore \quad T_{mars} = T_{mercury} \times \left(\frac{1.524}{0.387}\right)^{3/2}$$

$= (0.241\text{years}) \times (7.8)$

$= 1.9$ years.

Ex.20 If a graph is plotted between T^2 and r^3 for a planet then its slope will be -

(A) $\frac{4\pi^2}{GM}$ (B) $\frac{GM}{4\pi^2}$

(C) $4\pi\ GM$ (D) 0

Sol **(A)**

$$\frac{T^2}{r^3} = \frac{\left(\frac{2\pi r}{v_0}\right)^2}{r^3} = \frac{(2\pi r)^2}{r^3}\frac{1}{GM} r = \frac{4\pi^2}{GM}$$

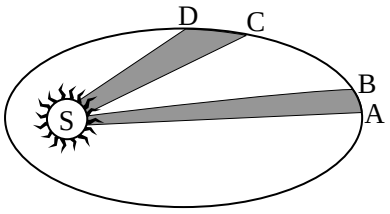
$$\left[\therefore \frac{mv_0^2}{r} = \frac{GMm}{r^2},\ v_0^2 = \frac{GM}{r}\right]$$

Slope of $T^2 - r^3$ curve $= \tan\theta$

$$= \frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

EXERCISE - 1

Q.1 In the figure, motion of a planet around the sun in elliptical path is shown with sun at one of its foci. Two shaded areas shown in figure are equal. If the times taken by the planet to move from A to B and from C to D are t_1 and t_2 respectively, then



(A) $t_1 < t_2$
(B) $t_1 > t_2$
(C) $t_1 = t_2$
(D) there is no relation between t_1 and t_2.

Q.2 Consider a particle moving in a circular orbit under the action of an attractive central force $F \propto \frac{1}{r}$, Its orbital period T will depend on r as -

(A) $T \propto r^{2/3}$ (B) $T \propto r$
(C) $T \propto r^{2/3}$ (D) $T \propto \frac{1}{r}$

Q.3 The loss in weight of a body taken from earth's surface to a height h is 1%. The change in weight taken into a mine of depth h will be -

(A) 1% loss (B) 1% gain
(C) 0.5% gain (D) 0.5 loss

Q.4 A body is released from a height equal to the radius R of earth. The velocity of the body with which it strikes the earth is -

(A) $\sqrt{gR}$ (B) $2\sqrt{gR}$
(C) $\sqrt{2gR}$ (D) $2\sqrt{2gR}$

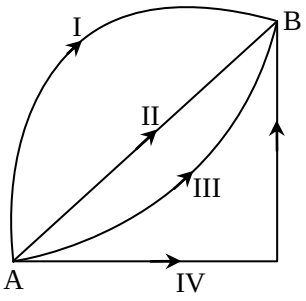
Q.5 The weight of a man is equivalent to 50kg-wt. If keeping the density of earth constant the radius of the earth is doubled, then the weight of the man will become -

(A) 100kg-wt (B) same
(C) half (D) zero

Q.6 Two satellites A and B (each of mass m) of earth are put in circular orbits around the centre of earth. The height of satellite A above the earth's surface is equal to the radius R of earth and that of satellite B is 3R. The ratio of potential energies of satellites A and B is -

(A) 1 : 2 (B) 2 : 1
(C) 3 : 1 (D) 1 : 3

Q.7 A particle is carried from A to B along different paths in a gravitational field as shown in figure, then

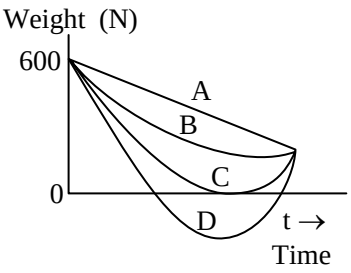


(A) Work done along path IV will be maximum
(B) Work done along path I will be maximum.
(C) Work done along all paths will be same
(D) Work done along path III will be minimum

Q.8 If the angular speed of earth is increased so much that the objects start flying from the equator, then the length of the day will be nearly-

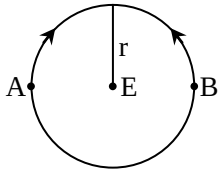
(A) $1\frac{1}{2}$ hours (B) 8 hours
(C) 18 hours (D) 24 hours

Q.9 Imagine the acceleration due to gravity on earth is $10 m/s^2$ and on mars is $4 m/s^2$. A traveller of mass 60kg goes from earth to mars by a rocket moving with constant velocity. If effect of other planets is assumed to be negligible, which one of the following graphs shown the variation of weight of traveller with time –



(A) A (B) B
(C) C (D) D

Q.10 Two satellites of same mass m are revolving round the earth E(mass M) in the same orbit of radius r. Rotational directions of the two are opposite therefore, they can collide and stick with each other. Total mechanical energy of the system (both satellites and earths) is - (m < < M)



(A) $-\frac{GMm}{r}$ (B) $-\frac{2GMm}{r}$
(C) $-\frac{GMm}{2r}$ (D) zero

Q.11 A missile which missed its target, went into orbit around the earth. The radius of the orbit is four times the radius of the parking orbit of a satellite. The period of the missile as satellite is -
(A) 2 days (B) 4 days
(C) 8 days (D) 16 days

Q.12 The acceleration due to gravity on the moon is one-sixth that on earth. If the average density of moon is three-fifth that of earth, the moon's radius in terms of earth's radius R_e is -
(A) $0.16R_e$ (B) $0.27R_e$
(C) $0.32R_e$ (D) $0.36R_e$

Q.13 The ratio of radii of two satellites is p and the ratio of their accelerations due to gravity is q. The ratio of their escape velocities will be -
(A) $\left(\frac{q}{p}\right)^{1/2}$ (B) $\left(\frac{p}{q}\right)^{1/2}$
(C) pq (D) $\sqrt{pq}$

Q.14 The acceleration due to gravity of that planet whose mass and radius are half those of earth, will be (g is acceleration due to gravity at earth's surface)
(A) 2g (B) g
(C) g/2 (D) g/4

Q.15 A satellite is revolving around earth in a circular orbit. The radius of orbit is half of the radius of the orbit of moon. Satellite will complete one revolution in –
(A) $2^{-3/2}$ lunar month
(B) $2^{-2/3}$ lunar month
(C) $2^{3/2}$ lunar month
(D) $2^{2/3}$ lunar month

Q.16 Assuming earth as a sphere of radius R, the weight of mass 1 kg at a distance 2R from the centre of earth is 2.5N. The weight of same mass at a distance 3R will be -
(A) 4.75N (B) 3.75N
(C) 2.5N (D) 1.1 N

Q.17 If a body is taken up to a height of 1600km from the earth's surface, then the percentage loss of gravitational force acting on that body will be - (Radius of earth R_e = 6400 km)
(A) 50% (B) 36%
(C) 25% (D) 10%

Q.18 The escape speed for a projectile in the case of earth is 11.2 km/sec. A body is projected from the surface of the earth with a velocity which is equal to twice the escape speed. The velocity of the body when at infinite distance from the centre of the earth is :
(A) 11.2 km/sec (B) 22.4 km/sec
(C) $11.2\sqrt{3}$ km/sec (D) $11.2\sqrt{2}$ km/sec

Q.19 Two identical satellites are moving in the same circular orbit around the earth but in opposite senses of rotation, Assuming an inelastic collision to take place, so that the wreckage remains as one piece of tangled material, the wreckage
(A) will be moving in a circular orbit of half the radius
(B) will be moving in a circular orbit on one fourth the radius
(C) will spin with no translatory motion
(D) falls directly down

Q.20 Consider the earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a balloon. The gravitational field measured by -
(A) A goes on decreasing and that by B goes on increasing
(B) B goes on decreasing and that by A goes on increasing
(C) each remains unchanged
(D) each goes on decreasing

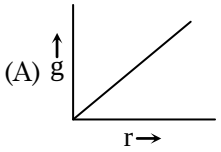
Q.21 An iron ball and a wooden ball of the same radius are released from a height h in vacuum. The times taken by both of these to reach the ground are -
(A) unequal (B) exactly equal
(C) roughly equal (D) zero

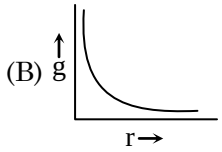
Q.22 A body of mass m is taken from the surface of earth to a height R/4 above the surface. The change in its potential energy is -
(A) $\frac{5mgR}{4}$ (B) $\frac{3mgR}{4}$
(C) $\frac{mgR}{4}$ (D) $\frac{mgR}{5}$

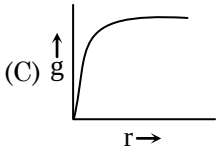
Q.23 An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is -
(A) $-E_0$ (B) $5E_0$
(C) $2E_0$ (D) E_0

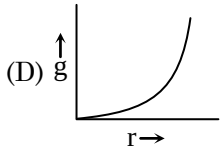
Q.24 The diameters of two planets are in ratio 4 : 1. Their mean densities have ratio 1 : 2. The ratio of 'g' on the planets will be :
(A) 1 : 2 (B) 1 : 4
(C) 2 : 1 (D) 4 : 1

Q.25 The weight of a man on earth is 600N. The acceleration due to gravity of moon is $\frac{1}{6}$th the acceleration due to gravity on earth. If the acceleration due to gravity on earth is 10 m/s^2. then the weight of the man on moon will be -
(A) 100 N (B) 300N
(C) 600N (D) 900N

Q.26 If the density of a planet is constant, then the curve between the g on its surface and its radius r will be -
(A) 

(B) 

(C) 

(D) 

Q.27 The mass of moon is $\frac{1}{81}$of the mass of earth and the acceleration due to gravity is $\frac{1}{6}$th of acceleration due to gravity on earth. The ratio of radii of moon and earth is -
(A) 6/81 (B) 4/729
(C) $\sqrt{2/27}$ (D) $\sqrt{2/48}$

Q.28 If a person can jump on the earth's surface upto a height of 2m, his jump on a satellite where acceleration due to gravity is 1.96 m/s^2, will be -
(A) 5m (B) 10m
(C) 20m (D) 2m

Q.29 If acceleration due to gravity at any point is g/2, then the intensity of gravitational field at that point will be -

(A) g/2 (B) 2g

(C) g (D) zero

Q.30 Two boys of masses 50kg and 60kg are 1m apart. The gravitational force of attraction between them

($G = 6.67 \times 10^{-11} N \times m^2/kg^2$) is -

(A) 2×10^{-10} N (B) 2×10^{-7} N

(C) 2×10^{-4} N (D) 2×10^{-1} N

Q.31 There are two bodies of masses 100 kg and 10000kg separated by a distance 1m. At what distance from the smaller body, the intensity of gravitational field will be zero ?

(A) $\frac{1}{9}$ m (B) $\frac{1}{10}$ m

(C) $\frac{1}{11}$ m (D) $\frac{10}{11}$ m

Q.32 Escape velocity of a 1kg body on a planet is 100m/s. Potential energy of body at that planet is

(A) – 5000 J (B) – 1000 J

(C) – 2400 J (D) – 10000 J

Q.33 The escape velocity for a rocket from earth is 11.2km/sec. Its value on a planet where acceleration due to gravity is double that on the earth and diameter of the planet is twice that of earth will be in km/sec

(A) 11.2 (B) 5.6

(C) 22.4 (D) 53.6

Q.34 The escape velocity from a planet is V. If its mass and radius becomes four and two times respectively, then the escape velocity will become

(A) V (B) 2V

(C) 0.5V (D) $V\sqrt{2}$

Q.35 A satellite is projected vertically upward with a velocity $\sqrt{1.5}$ times the orbital velocity just above the surface of earth. The maximum distance of the satellite from earth will be -

(A) $2R_e$ (B) $8R_e$

(C) $4R_e$ (D) $3R_e$

Q.36 The average radii of orbits of mercury and earth around the sun are 6×10^7km and 1.5×10^8 km respectively, The ratio of their orbital speeds will be -

(A) $\sqrt{5} : \sqrt{2}$ (B) $\sqrt{2} : \sqrt{5}$

(C) 2.5 : 1 (D) 1 : 25

Q.37 With what velocity a satellite is to be projected to establish it at a height 6R above the earth's surface -

($g = 9.8$ m/s^2 and $R = 6400$ km)

(A) 11.2 km/s (B) 10.4 km/s

(C) 8.6 km/s (D) 7.9 km/s

Q.38 Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-5/2}$, then T^2 is proportional to

(A) R^3 (B) $R^{7/2}$

(C) $R^{5/2}$ (D) $R^{3/2}$

EXERCISE - 2

Q.1 A geostationary satellite is revolving at a height 6R above the earth's surface, where R is the radius of earth. The period of revolution of a satellite orbiting at a height 2.5R above the earth's surface will be -

(A) 24 hours (B) 12 hours

(C) 6 hours (D) $6\sqrt{2}$ hours

Q.2 The value of g at any place is 9.8m/s^2. If the size of the earth suddenly shrinkes to half but the density remains unchanged, then the value of g at that place will be -

(A) 4.9 m/s^2 (B) 9.8 m/s^2

(C) 3.1 m/s^2 (D) 19.6 m/s^2

Q.3 How much energy will be needed for a body of mass 100kg to escape from the earth -

(g = 10m/s^2 and radius of earth = 6.4 x 10^{6}m)

(A) 6.4×10^9 joule (B) 8×10^6 joule

(C) 4×10^{16} joule (D) zero

Q.4 The acceleration due to gravity on the surface of earth is 10m/s^2 and the radius of earth is 6400km. With what minimum velocity must a body be thrown from the surface of the earth so that is reaches a height of 6400km?

(A) 8 km/s (B) 64km/s

(C) 1 km/s (D) 32 km/s

Q.5 The mass of planet mars is $\frac{1}{10}$ th of the mass of earth and radius is $\frac{1}{2}$ of the radius of earth. If the escape velocity at the earth is 11.2km/s, then the escape velocity at Mars will be -

(A) 10km/s (B) 5 km/s

(C) 20 km/s (D) 40 km/s

Q.6 If the radius of earth is 6400km and the acceleration due to gravity g is 10m/s^2, the angular velocity of earth for which the weight of a body would become zero at equator, will be -

(A) $\frac{1}{400}$ rad/s (B) $\frac{1}{800}$ rad/s

(C) $\frac{1}{1600}$ rad/s (D) impossible

Q.7 The speed of a satellite of mass 500kg at a height 10^3 km above the earth's surface is 7.36×10^3 m/s. If the orbit of satellite is circular then the gravitational force due to earth on satellite is (in N)

(A) 4×10^2 (B) 3.66×10^3

(C) 3.75×10^4 (D) 4.5×10^2

Q.8 Angular momentum of a planet of mass m orbiting around sun is J, a real velocity of its radius vector will be -

(A) $\frac{1}{2}$mJ (B) $\frac{J}{2m}$ (C) $\frac{m}{2J}$ (D) $\frac{1}{2mJ}$

Q.9 The potential energy of a body of mass m is

U = ax + by the magnitude of acceleration of the body will be -

(A) $\frac{ab}{m}$ (B) $\left(\frac{a+b}{m}\right)$

(C) $\frac{\sqrt{a^2+b^2}}{m}$ (D) $\frac{a^2+b^2}{m}$

Q.10 If the length of the day is T, the height of that TV satellite above the earth's surface which always appears stationary from earth, will be-

(A) $h = \left(\frac{4\pi^2 GM}{T^2}\right)^{1/3}$

(B) $h = \left(\frac{4\pi^2 GM}{T^2}\right)^{1/3} - R$

(C) $h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$

(D) $h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} + R$

Q.11 A number of particles each of mass 0.75kg are placed at distances 1m, 2m, 4m, 8m etc. from origin along positive X axis. the intensity of gravitational field at the origin will be -

(A) G (B) $\frac{3}{4}$G

(C) $\frac{1}{2}$G (D) zero

Q.12 What should be the angular velocity of earth about its own axis so that the weight of the body at the equator would become $\frac{3}{5}$th of its present value. (Radius of earth at equator = R_e)

(A) $\sqrt{\frac{5\ g}{2R_e}}$ (B) $\sqrt{\frac{2\ g}{5R_e}}$

(C) $\sqrt{\frac{3\ g}{5R_e}}$ (D) $\sqrt{\frac{5\ g}{3R_e}}$

Q.13 A satellite whose mass is m, is orbiting around the earth at a height R above the earth's surface. If the intensity of gravitational field at the earth's surface is g and radius of earth is R, then the kinetic energy of the satellite will be -

(A) mgR/4 (B) mgR/2

(C) mgR (D) 2 mgR

Q.14 The masses of moon and earth are 7.36×10^{22} kg and 5.98×10^{24} kg respectively and their mean separation is 3.82×10^5 km. The energy required to break the earth-moon system is -

(A) 12.4×10^{32} J (B) 7.68×10^{28}J

(C) 5.36×10^{24}J (D) 3.84×10^{28} J

Q.15 Three identical point masses, each of mass 1kg lie in the x-y plane at points (0, 0) (0, 0.2m) and (0.2m, 0). The gravitational force on the mass at the origin is -

(A) $1.67 \times 10^{-9}\ (\hat{i} + \hat{j})$ N

(B) $3.34 \times 10^{-10}\ (\hat{i} + \hat{j})$ N

(C) $1.67 \times 10^{-9}\ (\hat{i} - \hat{j})$ N

(D) $3.34 \times 10^{-10}\ (\hat{i} - \hat{j})$ N

Q.16 The value of g at any point is 9.8m/s². If earth is reduced to half of its present size by shrinking without loss of mass but the point remains at the same position, then the value of g at that point will be -

(A) 4.9 m/s² (B) 3.1 m/s²

(C) 9.8m/s² (D) 19.6 m/s²

Q.17 One planet is orbiting around the sun, At a point P it is at minimum distance d_1 from the sun and at that time speed is v_1. If at a another point Q it is at maximum distance d_2 from the sun then at this point the speed of planet will be -

(A) $d_1^2v_1 / d_2^2$ (B) d_2v_1/d_1

(C) d_1v_1/d_2 (D) $d_2^2v_1/d_1^2$

Q.18 The radius of a planet is 74×10^3 km. A satellite of this planet completes one revolution in 16.7 days. If the radius of the orbit of satellite is 27 times the radius of planet, then the mass of planet is

(A) 2.3×10^{27} kg (B) 2.3×10^{30} kg

(C) 2.3×10^{32} kg (D) 2.3×10^{42} kg

Q.19 If M_e is the mass of earth and M_m is the mass of moon ($M_e = 81M_m$). The potential energy of an object of mass m situated at a distance R from the centre of earth and r from the centre of moon, will be -

(A) $-GmM_m\left(\frac{R}{81}+r\right)\frac{1}{R^2}$

(B) $-GmM_e\left(\frac{81}{r}+\frac{1}{R}\right)$

(C) $-GmM_m\left(\frac{81}{R}+\frac{1}{r}\right)$

(D) $GmM_m\left(\frac{81}{R}-\frac{1}{r}\right)$

Q.20 A body placed at a distance R_0 from the centre of earth, starts moving from rest. The velocity of the body on reaching at the earth's surface will be

(R_e = radius of earth and M_e = mass of earth)

(A) $GM_e\left(\frac{1}{R_e}-\frac{1}{R_0}\right)$ (B) $2GM_e\left(\frac{1}{R_e}-\frac{1}{R_e}\right)$

(C) $GMe\sqrt{\frac{1}{R_e}-\frac{1}{R_0}}$ (D) $\sqrt{2GM_e\left(\frac{1}{R_e}-\frac{1}{R_0}\right)}$

Q.21 The masses and the radii of earth and moon are respectively M_e, R_e and M_m, R_m. The distance between the centres is r. At what minimum velocity a particle of mass m be projected from the mid point of the distance between their centres so that it may escape into space ?

(A) $\sqrt{\frac{4G}{r}(M_e + M_m)}$ (B) $\frac{4G}{r}\sqrt{(M_e + M_m)}$

(C) $\sqrt{\frac{2G}{r}(M_e + M_m)}$ (D) $\frac{2G}{r}\sqrt{(M_e + M_m)}$

Q.22 Three particles each of mass 100gm are brought from a very large distance to the vertices of an equilateral triangle whose side is 20cm in length. The work done will be -

(A) 0.33×10^{-11} joule

(B) -0.33×10^{-11} joule

(C) 1.0×10^{-11} joule

(D) -1.00×10^{-11} joule

Q.23 A tunnel is dug along the diameter of the earth. If a particle of mass m is situated in the tunnel at a distance x from the centre of earth then gravitational force acting on it, will be -

(A) $\frac{GM_em}{R_e^3}x$ (B) $\frac{GM_em}{R_e^2}$

(C) $\frac{GM_em}{x^2}$ (D) $\frac{GM_em}{(R_e + x)^2}$

Q.24 Four particles of masses m, 2m, 3m and 4m are kept in sequence at the corners of a square of side a. The magnitude of gravitational force acting on a particle of mass m placed at the centre of the square will be -

(A) $\frac{24m^2G}{a^2}$ (B) $\frac{6m^2G}{a^2}$

(C) $\frac{4\sqrt{2}Gm^2}{a^2}$ (D) zero

Q.25 If the change in the value of g at a height h above the surface of the earth is the same as at a depth x below it. When both x and h are much smaller than the radius of the earth. Then -

(A) x = h (B) x = 2h

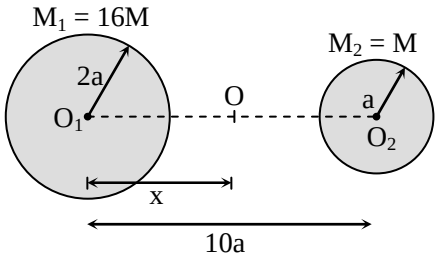
(C) x = h/2 (C) $x = h^2$

Q.26 The gravitational force on a body inside the surface of the earth varies as r^a, where r is the distance from the center of the earth and a is some constant. If the density of the earth is assumed uniform, then -

(A) a = 1 (B) a = – 1

(C) a = 2 (D) a = – 2

Q.27 Distance between the centres of two stars is 10a. The masses of these stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of G, M and a.



(A) $\upsilon_{min} = \frac{3}{2}\sqrt{\left(\frac{5GM}{a}\right)}$

(B) $\upsilon_{min} = \frac{5}{2}\sqrt{\left(\frac{5GM}{a}\right)}$

(C) $\upsilon_{min} = \frac{5}{2}\sqrt{\left(\frac{5M}{Ga}\right)}$

(D) None

EXERCISE - 3

Old Examination Questions [AIEEE/JEE Main]

Q.1 A mass m is raised from a distance 2 Re from surface of earth to 3Re. Work done to do so against gravity will be- **[AIEEE-2002]**

(A) $\frac{MgRe}{10}$ (B) $\frac{MgRe}{11}$

(C) $\frac{MgRe}{12}$ (D) $\frac{MgRe}{14}$

Q.2 The escape velocity of a body of mass m from earth depends on - **[AIEEE-2002]**

(A) m^2 (B) m^1

(C) m^0 (D) None of above

Q.3 If suddenly gravitational force on a satellite becomes zero it will – **[AIEEE-2002]**

(A) go in tangential direction of orbit
(B) fall on earth
(C) follow hellical path towards earth
(D) follow hellical path away from earth

Q.4 The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is – **[AIEEE-2002]**

(A) $\frac{mgR}{2}$ (B) 2 mgR

(C) mgR (D) $\frac{mgR}{4}$

Q.5 The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be– **[AIEEE-2003]**

(A) 22 km/s (B) 11 km/s

(C) $11/\sqrt{2}$ km/s (D) $11\sqrt{2}$

Q.6 The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become – **[AIEEE-2003]**

(A) 80 hours (B) 40 hours

(C) 20 hours (D) 10 hours

Q.7 A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[AIEEE-2004]**

(A) gx (B) $\frac{gR}{R-x}$

(C) $\frac{gR^2}{R+x}$ (D) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

Q.8 The time period of an earth satellite in circular orbit is independent of – **[AIEEE-2004]**

(A) The mass of the satellite
(B) Radius of its orbit
(C) Both the mass and radius of the orbit
(D) Neither the mass of the satellite nor the radius of its orbit

Q.9 If 'g' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to the radius 'R' of the earth is- **[AIEEE-2004]**

(A) 2 mgR (B) 1/2 mgR

(C) 1/4 mgR (D) mgR

Q.10 Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to – **[AIEEE-2004]**

(A) $R^{\left(\frac{n+1}{2}\right)}$ (B) $R^{\left(\frac{n-1}{2}\right)}$

(C) R^n (D) $R^{\left(\frac{n-2}{2}\right)}$

Q.11 Average density of the earth **[AIEEE-2005]**

(A) does not depend on g
(B) is a complex function of g
(C) is directly proportional to g
(D) is inversely proportional to g

Q.12 The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct ? **[AIEEE-2005]**

(A) $d = \frac{h}{2}$ (B) $d = \frac{3h}{2}$

(C) $d = 2h$ (D) $d = h$

Q.13 A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere (you may take $G = 6.67 \times 10^{-11}$ Nm^2 / kg^2) **[AIEEE-2005]**

(A) 13.34×10^{-10} J (B) 3.33×10^{-10} J

(C) 6.67×10^{-9} J (D) 6.67×10^{-10} J

Q.14 A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[AIEEE-2008]**

(A) 11 km s^{-1} (B) 110 km s^{-1}

(C) 0.11 km s^{-1} (D) 1.1 km s^{-1}

Q.15 The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is – **[AIEEE-2009]**

(A) $\frac{R}{\sqrt{2}}$ (B) R/2

(C) $\sqrt{2}$ R (D) 2R

Q.16 Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is : **[AIEEE-2011]**

(A) Zero (B) $-\frac{4Gm}{r}$

(C) $-\frac{6Gm}{r}$ (D) $-\frac{9Gm}{r}$

Q.17 Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is - **[AIEEE-2011]**

(A) $\sqrt{\frac{Gm}{4R}}$ (B) $\sqrt{\frac{Gm}{3R}}$

(C) $\sqrt{\frac{Gm}{2R}}$ (D) $\sqrt{\frac{Gm}{R}}$

Q.18 The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be- **[AIEEE-2012]**

(A) 6.4×10^8 Joules (B) 6.4×10^9 Joules

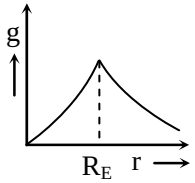
(C) 6.4×10^{10} Joules (D) 6.4×10^{11} Joules

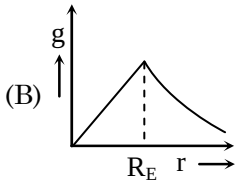
Q.19 A point particle is held on the axis of a rig of mass 'm' and radius 'r' at a distance 'r' from its centre 'c'. When released, it reaches 'c' under the gravitational attraction of the ring. it speed at 'c' will be : **[AIEEE Online-2012]**

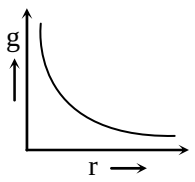
(A) $\sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$ (B) $\sqrt{\frac{Gm}{r}}$

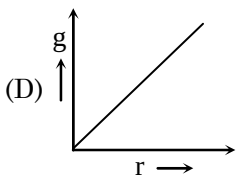
(C) $\sqrt{\frac{2Gm}{r}}$ (D) $\sqrt{\frac{2Gm}{r}}\left(\sqrt{2}-1\right)$

Q.20 Which graph correctly presents the variation of acceleration due to gravity with the distance from the centre of the earth (radius R_E) ? **[AIEEE Online-2012]**

(A) 

(B) 

(C) 

(D) 

Q.21 Assuming the earth to be a sphere of uniform density, the acceleration due to gravity inside the earth at a distance of r from the centre is proportional to :

[AIEEE Online-2012]

(A) r^{-2} (B) r^2

(C) r^{-1} (D) r

Q.22 What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R ?

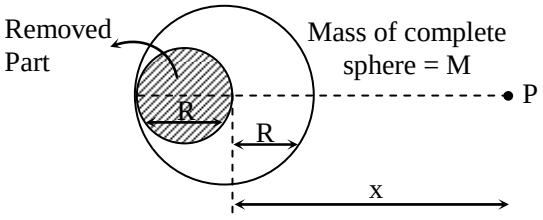
[JEE Main-2013]

(A) $\frac{GmM}{2R}$ (B) $\frac{GmM}{3R}$

(C) $\frac{5GmM}{6R}$ (D) $\frac{2GmM}{3R}$

Q.23 The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown has been 'removed out'), at a very far off point, P, located as shown, would be (nearly) :

[JEE Main Online-2013]



(A) $\frac{5}{6}\frac{GM}{x^2}$ (B) $\frac{8}{9}\frac{GM}{x^2}$

(C) $\frac{7}{8}\frac{GM}{x^2}$ (D) $\frac{6}{7}\frac{GM}{x^2}$

Q.24 The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is - (mass of the moon = 7.36×10^{22} kg, radius of the moon's orbit -= 3.8×10^8 m).

[JEE Main Online-2013]

(A) $6.73\times10^{-5}\frac{m}{s^2}$ (B) $6.73\times10^{-3}\frac{m}{s^2}$

(C) $6.73\times10^{-2}\frac{m}{s^2}$ (D) $6.73\times10^{-4}\frac{m}{s^2}$

Q.25 The gravitational filed in a region is given by

$$\vec{E} = (5N/kg)\hat{i} + (12N/kg)\hat{j}$$

If the potential at the origin is taken to be zero, then the ratio of the potential at the points (12m, 0) and (0, 5m) is-

[JEE Main Online-2013]

(A) Zero (B) 1

(C) $\frac{144}{25}$ (D) $\frac{25}{144}$

Q.26 Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is -

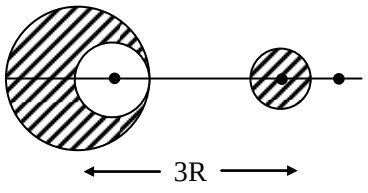
[JEE Main-2014]

(A) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (B) $\sqrt{\frac{GM}{R}(1+2\sqrt{2}}$

(C) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (D) $\sqrt{\frac{GM}{R}}$

Q.27 From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its center and the periphery .(See figure). For the configuration in the figure where the distance between the center of the original sphere and the removed sphere is 3R, the gravitational force between the two sphere is -

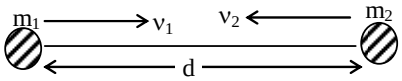
[JEE Main Online-2014]



(A) $\frac{41GM^2}{3600R^2}$ (B) $\frac{41GM^2}{450R^2}$

(C) $\frac{59GM^2}{450R^2}$ (D) $\frac{GM^2}{225R^2}$

Q.28



Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Beacause of the gravitational force they move towards each other along the line joining their centers. What is their speed when their separation is 'd' ?

[JEE Main Online-2014]

(Speed of m_1 is v_1 and that m_2 is v_2)

(A) $v_1 = v_2$

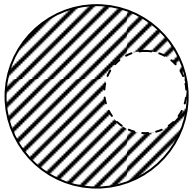
(B) $v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$ $v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

(C) $v_1 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$ $v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

(D) $v_1 = m_2\sqrt{\frac{2G}{m_1}}$ $v_2 = m_1\sqrt{\frac{2G}{m_2}}$

Q.29 From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is – **[JEE Main -2015]**

(G = gravitational constant)



(A) $\frac{-GM}{2R}$ (B) $\frac{-GM}{R}$

(C) $\frac{-2GM}{3R}$ (D) $\frac{-2GM}{R}$

Q.30 A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increases in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

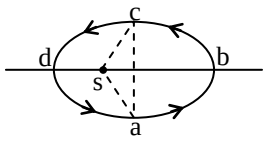
[JEE Main 2016]

(A) $\sqrt{2gR}$ (B) $\sqrt{gR}$

(C) $\sqrt{gR/2}$ (D) $\sqrt{gR}(\sqrt{2}-1)$

Q.31 Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) with db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then

[JEE Main Online 2016]



(A) $t_1 = t_2$ (B) $t_1 = 2t_2$

(C) $t_1 = 3t_2$ (D) $t_1 = 4t_2$

Q.32 An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is :

[JEE Main Online 2016]

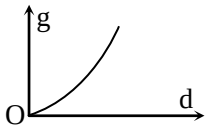
(A) Zero since astronaut feels weightless

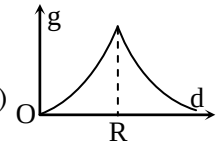
(B) $F_G = \frac{GMm}{(R+h)^2}$

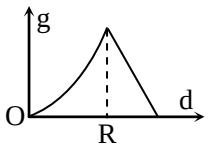
(C) $0 < F_G < \frac{GMm}{R^2}$

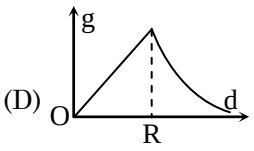
(D) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

Q.33 The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius)- **[JEE Main -2017]**

(A) 

(B) 

(C) 

(D) 

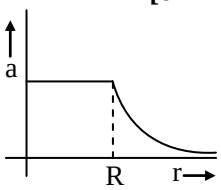
Q.34 If the earth has no rotational motion, the weight of a person on the equation is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weight $\frac{3}{4}$W. Radius of the earth is 6400 km and g = 10 m/s².

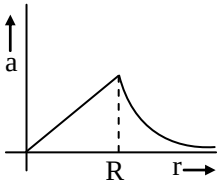
[JEE Main Online 2017]

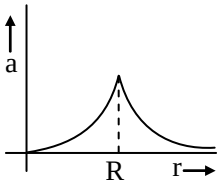
(A) 0.63×10^{-3} rad/s (B) 0.28×10^{-3} rad/s
(C) 1.1×10^{-3} rad/s (D) 0.83×10^{-3} rad/s

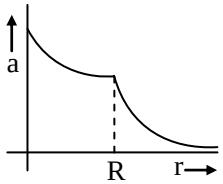
Q.35 The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \le R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a, of a test particle as a function of r is –

[JEE Main Online 2017]

(A)


(B)


(C)


(D)


Q.36 A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then - **[JEE Main - 2018]**

(A) $T \propto R^{3/2}$ for any n (B) $T \propto R^{\frac{n}{2}+1}$
(C) $T \propto R^{(n+1)/2}$ (D) $T \propto R^{n/2}$

Q.37 A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius R/2, and the other mass, in a circular orbit of radius 3R/2. The difference between the final and initial total energies is:

[JEE-Main Online-2018]

(A) $-\frac{GMm}{2R}$ (B) $+\frac{GMm}{6R}$
(C) $-\frac{GMm}{6R}$ (D) $\frac{GMm}{2R}$

Q.38 Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. The, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is: **[JEE-Main Online-2018]**

(A) 2 (B) 6
(C) 10^{-2} (D) 0.5

Q.39 Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence.

[JEE-Main Online-2018]

(A) There will be no change in weight anywhere on the earth
(B) Weight of the object, everywhere on the earth, will decrease
(C) Weight of the object, everywhere on the earth, will increase
(D) Except at poles, weight of the object on the earth will decrease

Q.40 The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mass of the earth I –

[JEE-Main Online-2018]

(A) 3×10^{-2} (B) 10^{-2}
(C) 2×10^{-2} (D) 6×10^{-2}

EXERCISE - 4

Old Examination Questions [IIT JEE Advanced]

Q.1 The magnitudes o the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then **[IIT - 1994]**

(A) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(B) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$ if $r_1 > R$ and $r_2 > R$

(C) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(D) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

Q.2 If the distance between the earth and the sun were half its present value, the number of days in a year would have been **[IIT - 1996]**

(A) 64.5 (B) 129

(C) 182.5 (D) 730

Q.3 A particle is projected vertically upward from the surface of earth (radius R_e) with a kinetic energy equal to half of the minimum vlaue needed for it to escape. The height to which it rises above the surface of earth is – **[IIT- 1997]**

(A) $h = R_e$ (B) $h = 2R_e$

(C) $h = R_e/2$ (D) $h = 3R_e$

Q.4 An artificial satellite moving in a circular orbit around the Earth has a total (kinetic, + potential) energy E_0. Its potential energy is- **[IIT- 1997]**

(A) $-E_0$ (B) $1.5\ E_0$

(C) $2\ E_0$ (D) E_0

Q.5 The ratio of Earth's orbital angular momentum (about the Sun) to its mass is 4.4×10^{15} m^2/s. The area enclosed by Earth's orbit is approximately. **[IIT- 1997]**

(A) 6.94×10^{22} m^2 (B) 6.94×10^{20} m^2

(C) 6.94×10^{18} m^2 (D) 6.94×10^{24} m^2

Q.6 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. **[IIT -1998]**

(A) The acceleration of S is always directed towards the centre of the earth

(B) The angular momentum of about the centre of the earth changes in direction , but its magnitude remains constant

(C) The total mechanical energy of S varies periodically with time

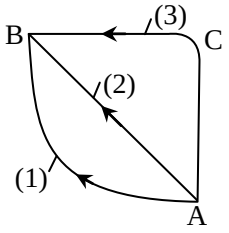
(D) The linear momentum of remains constant in magnitude

Q.7 A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6400 km) will approximately be **[IIT - 2002]**

(A) 1/2 hr (B) 1 hr

(C) 2 hr (D) 4 hr

Q.8 In a region of only gravitational field a particle of mass 'M' is shifted from A to B via three different paths as shown in the figure. The work done in different paths are W_1, W_2, W_3 respectively then **[IIT - 2003]**

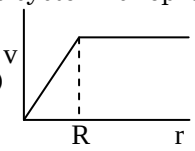


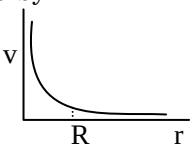
(A) $W_1 > W_2 > W_3$ (B) $W_1 = W_2 = W_3$

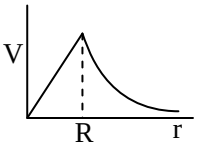
(C) $W_3 > W_2 > W_1$ (D) $W_2 > W_3 > W_1$

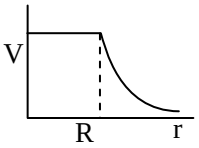
Q.9 A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$ where ρ_0 is a constant.

A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

Q.10 STATEMENT – 1 **[IIT - 2008]**

An astronaut in an orbiting space station above the Earth experiences weightlessness. **and**

STATEMENT – 2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.

(A) STATEMENT–1 is True,

STATEMENT- 2 is True;

STATEMENT-2 is a correct explanation for STATEMENT -1

(B) STATEMENT–1 is True,

STATEMENT- 2 is True;

STATEMENT-2 is **NOT** a correct explanation for STATEMENT -1

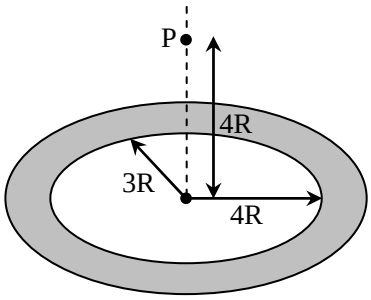
(C) STATEMENT-1 is True,

STATEMENT-2 is False

(D) STATEMENT-1 is False,

STATEMENT-2 is True

Q.11 A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is - **[IIT-2010]**



(A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

Q.12 Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}$g, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km/s, the escape speed on the surface of the planet in km/s will be ? **[IIT-2010]**

(A) 1 km/s (B) 2 km/s

(C) 3 km/s (D) 4 km/s

Q.13 A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass '*m*' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **[IIT-2011]**

(A) $\frac{1}{2}mV^2$ (B) mV^2

(C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Q.14 Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is ($M_P + M_Q$). The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R respectively. Then. **MCQ** **[IIT-2012]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$

(C) $V_R / V_P = 3$ (D) $V_P / V_Q = \frac{1}{2}$

Q.15 Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) -

MCQ [[JEE-Advanced 2013]

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant

Q.16 A planet of radius R = $\frac{1}{10}$ × (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and acceleration due to gravity on Earth is 10 ms^{-2}) - **[JEE-Advanced 2014]**

(A) 96 N (B) 108 N (C) 120 N (D) 150 N

Q.17 A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the *Sun-Earth system* is closest to (*Ignore the rotation and revolution of the Earth and the presence of any other planet*)

[JEE-Advanced 2017]

(A) $v_s = 42$ km s^{-1} (B) $v_s = 62$ km s^{-1}

(C) $v_s = 22$ km s^{-1} (D) $v_s = 72$ km s^{-1}

Q.18 The potentiel energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ?

[JEE-Advanced-2018]

(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$

(C) $L = \sqrt{mk}\,R^2$ (D) $L = \frac{\sqrt{mk}}{2}R^2$

Q.19 A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II **[JEE-Advanced-2018]**

LIST-I		LIST-II	
P.	$\frac{v_1}{v_2}$	**1.**	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	**2.**	1
R.	$\frac{K_1}{K_2}$	**3.**	2
S.	$\frac{T_1}{T_2}$	**4.**	8

(A) P → 4 ; Q → 2 ; R → 1 ; S → 3

(B) P → 3 ; Q → 2 ; R → 4 ; S → 1

(C) P → 2 ; Q → 3 ; R → 1 ; S → 4

(D) P → 2 ; Q → 3 ; R → 4 ; S → 1

ANSWER KEY

EXERCISE - 1

1. (C)	**2.** (B)	**3.** (D)	**4.** (A)	**5.** (A)	**6.** (B)	**7.** (C)
8. (A)	**9.** (C)	**10.** (B)	**11.** (C)	**12.** (B)	**13.** (D)	**14.** (A)
15. (A)	**16.** (D)	**17.** (B)	**18.** (C)	**19.** (D)	**20.** (D)	**21.** (B)
22. (D)	**23.** (C)	**24.** (C)	**25.** (A)	**26.** (A)	**27.** (C)	**28.** (B)
29. (A)	**30.** (B)	**31.** (C)	**32.** (A)	**33.** (C)	**34.** (D)	**35.** (D)
36. (A)	**37.** (B)	**38.** (B)				

EXERCISE - 2

1. (D)	**2.** (A)	**3.** (A)	**4.** (A)	**5.** (B)	**6.** (B)	**7.** (B)
8. (B)	**9.** (C)	**10.** (C)	**11.** (A)	**12.** (B)	**13.** (A)	**14.** (D)
15. (A)	**16.** (C)	**17.** (C)	**18.** (A)	**19.** (C)	**20.** (D)	**21.** (A)
22. (D)	**23.** (A)	**24.** (C)	**25.** (B)	**26.** (A)	**27.** (A)	

EXERCISE - 3

1. (C)	**2.** (C)	**3.** (A)	**4.** (C)	**5.** (B)	**6.** (B)	**7.** (D)
8. (A)	**9.** (B)	**10.** (A)	**11.** (A)	**12.** (C)	**13.** (D)	**14.** (B)
15. (D)	**16.** (D)	**17.** (A)	**18.** (C)	**19.** (A)	**20.** (B)	**21.** (D)
22. (C)	**23.** (C)	**24.** (A)	**25.** (B)	**26.** (C)	**27.** (A)	**28.** (B)
29. (B)	**30.** (D)	**31.** (C)	**32.** (B)	**33.** (D)	**34.** (A)	**35.** (A)
36. (C)	**37.** (C)	**38.** (A)	**39.** (D)	**40.** (C)		

EXERCISE - 4

1. (A)	**2.** (B)	**3.** (A)	**4.** (C)	**5.** (A)	**6.** (A)	**7.** (C)
8. (B)	**9.** (C)	**10.** (A)	**11.** (A)	**12.** (C)	**13.** (B)	**14.** (B,D)
15. (B,D)	**16.** (B)	**17.** (A)	**18.** (B,C)	**19.** (B)		

All exercise's detailed solutions are available at www.careerpoint.ac.in

SOLUTIONS

EXERCISE-4

1.[A] If $r < R$ $F \propto r$

$$\frac{F_1}{F_2} = \frac{r_1}{r_2}$$

If $r > R$ $F \propto \frac{1}{r^2}$

$$\frac{F_1}{F_2} = \frac{(r_2)^2}{(r_1)^2}$$

2.[B] $$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\frac{365}{T_2} = \left(\frac{r}{r/2}\right)^{3/2} \Rightarrow T_2 = 129 \text{ days}$$

3.[A] $$\frac{1}{2}\left(\frac{1}{2}m(V_e)^2\right) = \frac{mg_0h}{1+\frac{h}{R}}$$

$$\frac{1}{4}(2g_0R) = \frac{g_0h}{1+\frac{h}{R}}, \quad \frac{R}{2}+\frac{h}{2} = h$$

$h = R$

4.[C] Relation between KE, PE & E

$$E = \frac{PE}{2} = -KE, \quad PE = 2E = 2E_0$$

5.[A] $$\frac{dA}{dt} = \frac{J}{2m}$$

$$\text{Area} = \left(\frac{J}{2m}\right) \times 365 \times 86400$$

$$= \frac{1}{2}(4.4\times10^{15}) \times 365 \times 86400$$

$$= 6.94 \times 10^{22}$$

6.[A] (A) Acceleration of satellite towards the centre of earth

(B) Angular momentum always remains constant because external torque is zero

(C) Total mechanical energy is constant, gravitational force is a conservative force

(D) Linear momentum change because direction of velocity changes

7.[C] Time period of satellite orbiting near the surface of earth will be approximately 2 hr.

8.[B] W/D under the effect of conservative force does not depends on the path travelled by the body.

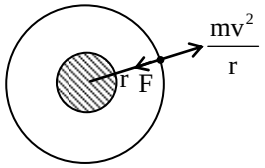
9.[C] If $r < R$ $F \propto r$

$F = kr$

$$F = \frac{mv^2}{r}$$

$$kr = \frac{mv^2}{r}$$

$v \propto r$

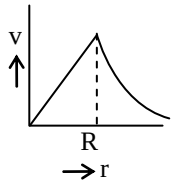


If $r > R$ $F \propto \frac{1}{r^2}$

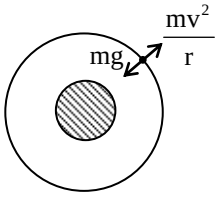
$$F = \frac{k}{r^2}$$

$$\frac{k}{r^2} = \frac{mv^2}{r}$$

$$v^2 \propto \frac{1}{r}$$



10.[A] Weight of astronaut is balanced by gravitational force which is also called state of free fall.



MOLE CONCEPT

JEE MAIN SYLLABUS

1. *Matter and its nature, Dalton's atomic theory.*
2. *Concept of atom, molecule, element and compound.*
3. *Physical quantities and their measurement in Chemistry, S.I. Units, dimensional analysis.*
4. *Laws of chemical combination.*
5. *Atomic and molecular masses, mole concept, molar mass, percentage composition, empirical and molecular formulae.*
6. *Chemical equation and stoichiometry.*

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Exercise-1		
Exercise-2		
Exercise-3		
Exercise-4		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Step-3: Solve Questions of Column B

Step-4: Solve questions from other Question Bank, Problem book etc.

Mole Concept

KEY CONCEPT

1. Significant Figures

(a) Every scientific observation involves some degree of uncertainty depending upon the limitation of instrument. To represent scientific data, role of significant figures has its own importance.

(b) Significant figures are equal to the number of digits in numbers with last digit uncertain and rest all are certain digits i.e. all the digits of datum including the uncertain one , are called significant figures.

(c) **Rules for determination significant figure:**

(i) All non zero digits are significant.

Example : 3.14 has three significant figures

(ii) The zeros to the right of the decimal point are significant.

Example : 3.0 has two significant figures.

(iii) The zeros to the left of the first non zero digit in a number are not significant.

Example : 0.02 has one significant figure.

(iv) The zeros between two non zero digits are also significant.

Example : 6.01 has three significant figures.

(v) **Exponential form :** $N \times 10^n$. Where N show the significant figure.

Example : 1.86×10^4 has three significant figure.

(vi) **Rounding off the uncertain digit :**

(1) If the left most digit to be rounded off is more than 5 , the preceding number is increased by one.

Example : 2.16 is rounded to 2.2

(2) If the left most digit to be rounded off is less than 5 , the preceding number is retained.

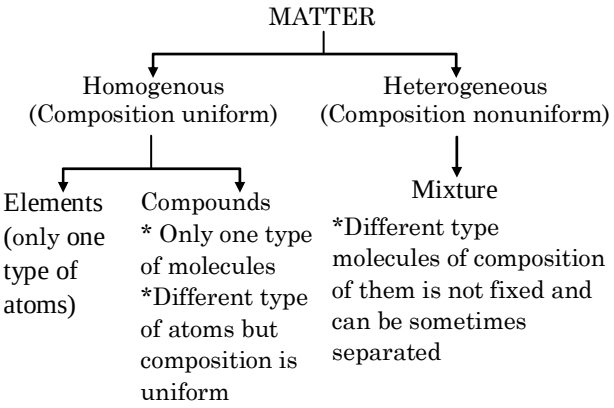
Example : 2.14 is rounded off to 2.1

(3) If the left most digit to be rounded off is equal to 5, the preceding number is not changed if it is even and increased by one if it is odd.

Example : 3.25 is rounded off to 3.2

2.35 is round off to 2.4

2. Types of Mixture



2.1 Heterogenous mixturez

A mixture in which the different constituents are not distributed uniformly is known as heterogenous mixture. **eg Water**

2.2 Homogenous mixture

A mixture in which the different constituents are uniformly distributed is known as homogenous mixture. eg. O_2, N_2 etc.

3. Laws of Chemical Combination

3.1 Law of conservation of mass-

[Lavoisier, 1744]

(i) According to this law , matter is neither created nor destroyed in the course of chemical reaction although it may change from one form to other

(ii) This law contradicts nuclear reactions where Einstein equation is applicable

(iii) According to this law , sum of the masses of product formed is always equal to the sum of the masses of the reactant undergone change

Example : $H_2 + Cl_2 \rightarrow 2HCl$

$\underbrace{2gm \quad 71\,gm}$ $\quad 2\,[1 + 35.5]$

$\Rightarrow \quad 73\ gm \quad \equiv \quad 73\ gm$

3.2 Law of definite proportion [Proust, 1799]

(i) According to the law, the composition of a compound always remains a constant i.e. the ratio of weights of different elements in a compound ; no matter by whatever method , it is prepared or obtained from different sources, remains always a constant

Example : In H_2O ratio of weight = 1 : 8

In CO_2 ratio of weight = 3 : 8

3.3 Law of multiple proportion

[John Dalton, 1804]

According to this law, when two elements A and B combine to form more than one chemical compound then different weights of A , which combine with a fixed weight of B , are in a proportion of simple whole number

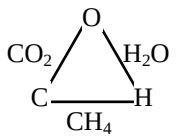
Example :	CO	&	CO_2
	12 : 16	&	12 : 32
ratio	=	16 : 32	
	=	1 : 2	

3.4 Law of reciprocal proportions

[Ritche, 1792-94]

When two elements combines separately with third element and form different types of molecules, their combining ratio is directly reciprocated if they combine directly

Example :



C with H form methane and with O form CO_2. In CH_4 , 12 grams of C reacts with 4 grams of H whereas in CO_2 12 gram of C reacts with 32 grams of O.

Therefore when H combines with O they should combine in the ratio of 4 : 32 (i.e. = 1 : 8) or in simple multiple of it. The same is found to be true in H_2O molecule. The ratio of weights of H and O in Water is 1 : 8

3.5 Gay-Lussac's [1808] law of combining volumes :

This law states that under similar conditions of pressure and temperature, volume ratio of gases is always in terms of simple integers.

Ex. $N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$

vol. ratio 1 : 3 : 2

4. Avogadro's Hypothesis

According to this under similar conditions of pressure and temperature , equal volumes of gases contain equal number of molecules.

4.1 Salient features of Avogadro's hypothesis

(i) It has removed the anomaly between Dalton's atomic theory and Gay Lussac's law of volume by making a clear distinction in between atoms and molecules

(ii) It reveals that common elementary gases like hydrogen , nitrogen , oxygen etc. are diatomic

(iii) It provides a method to determine the atomic weights of gaseous elements

(iv) It provides a relationship between vapour density and molecular weight of substances

Vapour density = $\dfrac{\text{Volume of definite amount of Gas}}{\text{Volume of same amount of Hydrogen}}$

or Vapour density = $\dfrac{\text{Weight of n molecules of Gas}}{\text{Weight of n molecules of Hydrogen}}$

or Vapour density = $\dfrac{\text{Weight of one molecule of Gas}}{\text{Weight of one atom of hydrogen} \times 2}$

or Vapour density = $\frac{\text{Molecular weight}}{2}$

(v) It helps to determine molar volume

Molecular weight of the gas

= 2 × vapour density

$= 2 \times \frac{\text{Weight of 1 litre of the Gas at S.T.P}}{\text{Weight of 1 litre of Hydrogen at S.T.P}}$

$= 2 \times \frac{\text{Weight of 1 litre of the Gas at S.T.P}}{0.089\,\text{gm}}$

$= \frac{2}{0.089} \times$ Weight of 1 litre of the gas at S.T.P.

= 22.4 × Weight of 1 litre of gas at S.T.P.

= Weight of 22.4 litre of the gas at S.T.P

5. Atom, Molecules and Molecular Formula

Atom: It is the smallest particle of an element that takes part in a chemical reaction and not capable of independent existence.

Molecule : It is the smallest particle of matter which is capable of independent existence. A molecule is generally an assembly of two or more tightly bonded atoms.

Homoatomic molecules : Molecules of an element contain one type of atoms. eg. O_2, Cl_2 etc.

Heteroatomic molecules : Molecules of compounds contain more that one type of atom.

eg. H_2O, HCl etc

5.1 Atomic mass scale

5.1.1 Oxygen as standard : The standard reference for atomic weight may be oxygen with an assigned value of 16.

Atomic weight of an element =

$$\frac{\text{Weight of 1 atom of element}}{1/16 \times \text{Weight of 1 atom of oxygen}}$$

5.1.2 Carbon as standard : The modern reference standard for atomic weight is carbon isotope of mass number 12.

Atomic weight of an element =

$$\frac{\text{Weight of 1 atom of the element}}{1/12 \times \text{Weight of 1 atom of C} - 12}$$

IMPORTANT POINTS

- Atomic weight is not a weight but a number.
- Atomic weight is not absolute but relative to the weight of the standard reference element C-12

5.2 Molecular weight

It is the number of times a molecule is heavier than 1/12th of an atom of C – 12.

$$\text{Molecular weight} = \frac{\text{Weight of 1 molecule}}{1/12 \times \text{Weight of one C} - 12}$$

IMPORTANT POINTS

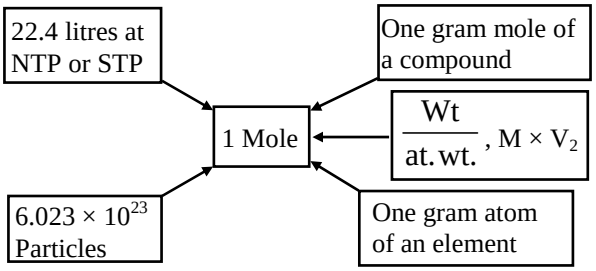
- Molecular weight is not a weight but a number
- Molecular weight is relative and not absolute
- Molecular weight expressed in grams is called gram molecular weight
- Molecular weight is calculated by adding all the atomic weights of all the atoms in a molecule

Example : $CO_2 = 12 + 2 \times 16 = 44$

6. Mole Concept

One mole is the amount of a substance that contains as many particles or entities as there are atoms in exactly

12gm of the 12_C isotope



IMPORTANT :

⇒ 1 mole = 6.023×10^{23} particles

⇒ 1 mole atoms = 6.023×10^{23} atoms

⇒ One mole molecule =

6.023×10^{23} molecules

⇒ Mass of one mole of atoms

= Gram atomic mass

⇒ Mass of one mole of molecules

= Gram molecular mass

⇒ Moles of a compound = Mass of compound

⇒ Volume occupied by 1 mole of a gas at N.T.P = 22.4 litres.

7. Chemical Formula

It is of two types -

(a) Molecular formulae : Chemical formulae that indicate the actual numbers and type of atoms in a molecule are called molecular formulae

(b) Empirical formulae : The chemical formulae that give only the relative number of atoms of each type in a molecule are called empirical formulae

7.1 Determination of chemical formulae

7.1.1 Determination of empirical formulae :

Step - I : Determination of percentage

Step - II : Determination of mole ratio

Step - III : Making it whole number ratio

Step - IV : Removal of fractions from mole ratio

7.1.2 Determination of molecular formulae

(i) First of all find empirical formulae

(ii) Molecular formulae

= (Empirical formulae) n

$$\text{where } n = \frac{\textit{Molecular weight}}{\textit{Empirical formula weight}}$$

9. Chemical Equation

Representation of the chemical change in terms of symbol and formulae of the reactants & products is called a chemical equation.

8.1 Information conveyed by a chemical equation

(i) Qualitatively , a chemical equation tells us the names of the various reactants

(ii) Quantitatively , it express

(1) The relative no. of molecules of reactants and products

(2) The relative no. of moles of reactant and products

(3) The relative masses of reactants and products

(4) The relative volumes of gaseous reactants and products

8.2 Limitations of chemical equations

(i) The physical state of the reactants and products

(ii) The dilution of solution of reactants and products are in soluble state

(iii) The energy changes during chemical reaction

(iv) The conditions of P, T etc at which reaction occurs.

(v) The rate of chemical reaction

8.3 Limiting Reagent

It may be defined as the reactant which is completely consumed during the reaction is called limiting reagent-

Example :

$2H_2(g) + O_2(g) \rightarrow 2H_2O$

Here H_2 is known as limiting reagent.

SOLVED EXAMPLES

Ex.1 8 litre of H_2 and 6 litre of Cl_2 are allowed to react to maximum possible extent. Find out the final volume of reaction mixture. Suppose P and T remains constant throughout the course of reaction -

(A) 7 litre (B) 14 litre

(C) 2 litre (D) None of these.

Sol.(B)

	H_2 +	Cl_2 $\rightarrow$	2HCl
Vol. before reac.	8 lit	6 lit	0
Vol. after reac.	2	0	12

$\therefore$ Volume after reaction

= Volume of H_2 left + Volume of HCl formed

= 2 + 12 = 14 lit

Ex.2 Naturally occurring chlorine is 75 % Cl^{35} and 25 % Cl^{37}. Calculate the average atomic mass of chlorine-

(A) 35.5 amu

(B) 36.5 amu

(C) 71 amu

(D) 72 amu

Sol.(A) Average atomic mass

$$= \frac{\%\text{ of I isotope} \times \text{its at. mass} + \%\text{ of II isotope} \times \text{Its at. mass}}{100}$$

$$= \frac{(75 \times 35) + (25 \times 37)}{100} = 35.5 \text{ amu.}$$

Ex.3 Calculate the mass in gm of 2g atom of Mg-

(A) 12 gm (B) 24 gm

(C) 6 gm (D) None of these.

Sol.(D) $\because$ 1 gm atom of Mg has mass = 24 gm

$\therefore$ 2 gm atom of Mg has mass

= 24 x 2 = 48 gm.

Ex.4 Calculate the weight of one atom of Ag – (At. wt. of Ag = 108)

(A) 17.93×10^{-23}gm (B) 16.93×10^{-23} gm

(C) 17.93×10^{23} gm (D) 36×10^{-23} gm

Sol.(A) $\because$ N atoms of Ag weigh 108 gm

$\therefore$ 1 atom of Ag weigh = $\frac{108}{N}$

$$= \frac{108}{6.023 \times 10^{23}} = 17.93 \times 10^{-23} \text{ gm.}$$

Ex.5 In 5g atom of Ag (at. wt. = 108), calculate the no. of atoms of Ag -

(A) 1 N (B) 3N

(C) 5 N (D) 7 N

Sol.(C) $\because$ 1 gm atom of Ag has atoms = N

$\therefore$ 5 gm atom of Ag has atoms = 5N.

Ex.6 Calculate the mass in gm of 2N molecules of CO_2 -

(A) 22 gm

(B) 44 gm

(C) 88 gm

(D) None of these.

Sol.(C) $\because$ N molecules of CO_2 has

molecular mass = 44.

$\therefore$ 2N molecules of CO_2 has

molecular mass = $44 \times 2 = 88$ gm.

Ex.7 How many carbon atoms are present in 0.35 mol of $C_6H_{12}O_6$ -

(A) 6.023×10^{23} carbon atoms

(B) 1.26×10^{23} carbon atoms

(C) 1.26×10^{24} carbon atoms

(D) 6.023×10^{24} carbon atoms

Sol.(C) $\therefore$ 1 mol of $C_6H_{12}O_6$ has = 6 N atoms of C

$\therefore$ 0.35 mol of $C_6H_{12}O_6$ has

= 6×0.35 N atoms of C = 2.1 N atoms

= $2.1 \times 6.023 \times 10^{23}$

= 1.26×10^{24} carbon atoms

Ex.8 How many molecules are in 5.23 gm of glucose ($C_6H_{12}O_6$) -

(A) 1.65×10^{22} (B) 1.75×10^{22}

(C) 1.75×10^{21} (D) None of these

Sol.(B) $\because$ 180 gm glucose has = N molecules

$\therefore$ 5.23 gm glucose has = $\frac{5.23 \times 6.023 \times 10^{23}}{180}$

= 1.75×10^{22} molecules

Ex.9 What is the weight of 3.01×10^{23} molecules of ammonia -

(A) 17 gm

(B) 8.5 gm

(C) 34 gm
(D) None of these

Sol. (B) ∵ 6.023×10^{23} molecules of NH_3 has weight = 17 gm

∴ 3.01×10^{23} molecules of NH_3 has

weight = $\frac{17 \times 3.01 \times 10^{23}}{6.023 \times 10^{23}}$ = 8.50 gm

Ex.10 How many molecules are present in one mℓ of water vapours at STP -
(A) 1.69×10^{19} (B) 2.69×10^{-19}
(C) 1.69×10^{-19} (D) 2.69×10^{19}

Sol.(D) ∵ 22.4 litre water vapour at STP has
= 6.023×10^{23} molecules
∴ 1×10^{-3} litre water vapours at STP has
= $\frac{6.023 \times 10^{23}}{22.4} \times 10^{-3} = 2.69 \times 10^{+19}$

Ex.11 How many years it would take to spend Avogadro's number of rupees at the rate of 1 million rupees in one second -
(A) 19.098×10^{19} years
(B) 19.098 years
(C) 19.098×10^{9} years
(D) None of these

Sol.(C) ∵ 10^6 rupees are spent in 1sec.
∴ 6.023×10^{23} rupees are spent in
= $\frac{1 \times 6.023 \times 10^{23}}{10^6}$ sec
= $\frac{1 \times 6.023 \times 10^{23}}{10^6 \times 60 \times 60 \times 24 \times 365}$ years
= 19.098×10^9 year

Ex.12 An atom of an element weighs 6.644×10^{-23} g. Calculate g atoms of element in 40 kg-
(A) 10 gm atom
(B) 100 gm atom
(C) 1000 gm atom
(D) 10^4 gm atom

Sol.(C) ∵ weight of 1 atom of element
= 6.644×10^{-23} gm
∴ weight of 'N' atoms of element
= $6.644 \times 10^{-23} \times 6.023 \times 10^{23}$
= 40 gm
∵ 40 gm of element has 1 gm atom.
∴ 40×10^3 gm of element has $\frac{40 \times 10^3}{40}$
= 10^3 gm atom.

Ex.13 Calculate the number of Cl^- and Ca^{+2} ions in 222 g anhydrous $CaCl_2$ -
(A) 2N ions of Ca^{+2} 4 N ions of Cl^-
(B) 2N ions of Cl^- & 4N ions of Ca^{+2}
(C) 1N ions of Ca^{+2} & 1N ions of Cl^-
(D) None of these.

Sol.(A) ∵ mol. wt. of $CaCl_2$ = 111 g
∵ 111 g $CaCl_2$ has = N ions of Ca^{+2}
∴ 222g of $CaCl_2$ has $\frac{N \times 222}{111}$
= 2N ions of Ca^{+2}
Also ∵ 111 g $CaCl_2$ has = 2N ions of Cl^-
∴ 222 g $CaCl_2$ has $\frac{2N \times 222}{111}$ = ions of Cl^-
= 4N ions of Cl^- .

Ex.14 The density of O_2 at NTP is 1.429g / litre. Calculate the standard molar volume of gas-
(A) 22.4 lit.
(B) 11.2 lit
(C) 33.6 lit
(D) 5.6 lit.

Sol.(A) ∵ 1.429 gm of O_2 gas occupies vol.= 1 litre.
∴ 32 gm of O_2 gas occupies = $\frac{32}{1.429}$
= 22.4 litre/mol.

Ex.15 Which of the following will weight maximum amount-
(A) 40 g iron
(B) 1.2 g atom of N
(C) 1×10^{23} atoms of carbon
(D) 1.12 litre of O_2 at STP

Sol.(A) (A) Mass of iron = 40 g
(B) Mass of 1.2 g atom of N
= 14 × 1.2 = 16.8 gm
(D) Mass of 1×10^{23} atoms of C
= $\frac{12 \times 1 \times 10^{23}}{6.023 \times 10^{23}}$ = 1.99 gm.

(D) Mass of 1.12 litre of O_2 at STP

$$= \frac{32\times1.2}{22.4} = 1.6 \text{ g}$$

Ex.16 How many moles of potassium chlorate to be heated to produce 11.2 litre oxygen -

(A) $\frac{1}{2}$ mol (B) $\frac{1}{3}$ mol

(C) $\frac{1}{4}$ mol (D) $\frac{2}{3}$ mol.

Sol.(B) $2\ KClO_3 \rightarrow 2KCl + 3O_2$

Mole for reaction 2 2 3

$\because$ 3×22.4 litre O_2 is formed by 2 mol $KClO_3$

$\therefore$ 11.2 litre O_2 is formed by $\frac{2\times11.2}{3\times22.4}$

$= \frac{1}{3}$ mol $KClO_3$

Ex.17 Calculate the weight of lime (CaO) obtained by heating 200 kg of 95% pure lime stone ($CaCO_3$).

(A) 104.4 kg (B) 105.4 kg

(C) 212.8 kg (D) 106.4 kg

Sol.(D) $\because$ 100 kg impure sample has pure $CaCO_3 = 95$ kg

$\therefore$ 200 kg impure sample has pure $CaCO_3$

$= \frac{95\times200}{100} = 190$ kg.

$CaCO_3 \rightarrow CaO + CO_2$

$\because$ 100 kg $CaCO_3$ gives CaO = 56 kg.

$\therefore$ 190 kg $CaCO_3$ gives CaO = $\frac{56\times190}{100}$

$=$ **106.4 kg.**

Ex.18 The chloride of a metal has the formula MCl_3. The formula of its phosphate will be-

(A) M_2PO_4 (B) MPO_4

(C) M_3PO_4 (D) $M(PO_4)_2$

Sol.(B) MCl_3 indicates that metal is trivalent.

Ex.19 A silver coin weighing 11.34 g was dissolved in nitric acid. When sodium chloride was added to the solution all the silver (present as $AgNO_3$) was precipitated as silver chloride. The weight of the precipitated silver chloride was 14.35 g. Calculate the percentage of silver in the coin -

(A) 4.8 % (B) 95.2%

(C) 90 % (D) 80%

Sol.(B) $Ag + 2HNO_3 \rightarrow AgNO_3 + NO_2 + H_2O$

108

$AgNO_3 + NaCl \rightarrow AgCl + NaNO_3$

143.5

$\therefore$ 143.5 gm of silver chloride would be precipitated by 108 g of silver.

or 14.35 g of silver chloride would be precipitated 10.8 g of silver.

$\because$ 11.34 g of silver coin contain 10.8 g of pure silver.

$\therefore$ 100 g of silver coin contain $\frac{10.8}{11.34}\times 100$

$=$ **95.2%.**

Ex.20 If the Faraday were to be 60230 coulombs instead of 96500 coulombs, what will be the charge on an electron ?

Sol. One mole electron carries 1 Faraday charge.

As 6.023×10^{23} electrons carry = 60230 C

So 1 electron carries

$= \frac{60230}{6.023\times10^{23}}$ C = **1×10^{-19} C**.

Ex.21 On heating 0.199 g of a metallic oxide in a current of hydrogen 0.045 g of water is formed. Find the equivalent weight of the metal.

Sol. Weight of Metallic oxide = 0.199 g

As 16 g oxygen is present in = 18 g H_2O

So O_2 present in 0.045 g H_2O

$= \frac{16\times0.045}{18} = 0.04$ g

Weight of metal

$= 0.199 - 0.04 = 0.159$

Equivalent weight of metal

$= \frac{0.159\times8}{0.04}$

$=$ **31.8**

Ex.22 10 ml of 0.02 M $KMnO_4$ is required to oxidize 20 ml of oxalic acid of certain

strength. 25 ml of the same oxalic acid is required to neutralize 20 ml of NaOH of unknown strength. Find the amount of NaOH in a litre of the solution.
(Molecular weight of NaOH = 40) :

Sol. In acidic medium
1 M $KMnO_4$ = 5 N $KMnO_4$
0.02 M $KMnO_4$ = 0.1 N $KMnO_4$
According to normality equation,
N_1V_1 ($KMnO_4$) = N_2V_2 (Oxalic acid)
$0.1 \times 10 = N_2 \times 20$
$N_2 = \frac{10 \times 0.1}{20} = 0.05$ N
Again
N_1V_1 (Oxalic acid) = N_2V_2 (NaOH)
$0.05 \times 25 = N_2 \times 20$
$N_2 = \frac{0.05 \times 25}{20} = 0.0625$ N
As $S = N \times E$
So $S = 0.0625 \times 40$
$= \mathbf{2.5\ g\ L^{-1}}$.

Ex.23 What volume of a solution of hydrochloric acid containing 73 g of acid litre would suffice for the exact neutralization of sodium hydroxide obtained by allowing 0.46 g of metallic sodium to act upon water ?

Sol. $Na + H_2O \rightarrow NaOH + \frac{1}{2} H_2$
$NaOH + HCl \rightarrow NaCl + H_2O$
Thus, meq. of Na = Meq. of NaOH formed
= Meq. of HCl used
$\frac{0.46}{23} \times 1000 = \frac{73}{36.5} \times V$
(eq. of HCl = N × V) = **10 ml**

Ex.24 Find weight of iron which will be converted into its oxide by the action of 18 g of steam on it.

Sol. $3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2$
$3 \times 56 \quad 4 \times 18$
As by 72 g steam the weight of Fe oxidized = 168 g
So by 18 g " " " "
$= \frac{168 \times 18}{72} = 42$ g

Ex.25 2.68×10^{-3} moles of a solution containing an ion A^{n+} required 1.61×10^{-3} moles of MnO_4^- for the oxidation of A^{n+} to AO_3^- in an acidic medium. What is the value of n?

Sol. As 1.61×10^{-3} M $KMnO_4$
$\equiv 2.68 \times 10^{-3}$ M solution of A^{n+}
So M/5 $KMnO_4 = \frac{2.68 \times 10^{-3} M}{1.61 \times 10^{-3}} \times \frac{M}{5}$
= 0.33 M solution of A^{n+}
$0.33\ M = \frac{M}{5-n}$.
$5 - n = \frac{1}{0.33} = 3$
n = 2

Ex.26 The label on a H_2O_2 bottle reads as 10 Vol. Find the concentration of the H_2O_2%.

Sol. $2H_2O_2 \rightarrow 2H_2O + O_2$
10 vol. means 1 vol H_2O_2 = 10 ml O_2
1 g $H_2O_2 = \frac{22400}{68} = 329$ ml
1 litre of 10 vol. means = 10000 ml of O_2
Weight of H_2O_2 to give 10000 ml
$= \frac{1}{329} \times 10000 = 30.4$ g
Conc. = 30.4 g/lit
Conc. % = $30.4 \times \frac{100}{1000} = \mathbf{3.04\%}$

Ex.27 Calculate the volume of 20 g of hydrogen gas at NTP.

Sol. Moles of hydrogen gas $= \frac{20}{2} = 10$.
Volume of the gas at NTP
= no. of moles × 22.4
= 10 × 22.4
= **224 litres**.

Ex.28 Calculate the number of atoms of each element present in 122.5 g of $KClO_3$.

Sol. No. of moles of $KClO_3 = \frac{122.5}{122.5} = 1$.
(mol. wt. of $KClO_3$ = 122.5)
From the formula $KClO_3$, we know that 1 mole of $KClO_3$ contains 1 mole of K atoms, 1 mole of Cl atoms and 3 moles of O atoms.

$$\left.\begin{aligned} \therefore \text{ no. of atoms of K} &= 1\times 6.022\times 10^{23} \\ \text{no. of atoms of Cl} &= 1\times 6.022\times 10^{23} \\ \text{no. of atoms of O} &= 3\times 6.022\times 10^{23} \end{aligned}\right\}$$

Ex.29 If the components of air are N_2, 78%; O_2, 21% Ar, 0.9% and CO_2, 0.1% by volume what would be the molecular weight of air ?

Sol. The volume ratio of the gases will be the same as their mole ratio (Avogadro's principle)

$\therefore$ mol. wt. of air (wt. in g per mole)

$$= \frac{78\times 28+21\times 32+0.9\times 40+0.1\times 44}{78+21+0.9+0.1}$$

$$= 28.964$$

($N_2 = 28$, $O_2 = 32$, Ar = 40 and $CO_2 = 44$)

Ex.30 The vapour density (hydrogen = 1) of a mixture consisting of NO_2 and N_2O_4 is 38.3 at 26.7ºC. Calculate the number of moles of NO_2 in 100 g of the mixture.

Sol. Wt. of NO_2 = x g.

$$\therefore \text{ obs. mol. wt. (wt./mole)} = \frac{\text{wt. in g}}{\text{total moles}}$$

$$= \frac{100}{\left(\frac{x}{46}+\frac{100-x}{92}\right)} = 2\times 38.3. \quad (0.437 \text{ mole})$$

EXERCISE-1

Q.1 Which one of the following properties of an element is not variable ?
(A) Valency
(B) Equivalent mass
(C) Atomic mass
(D) All the three

Q.2 An element A is tetravalent and another element B is divalent. The formula of the compound formed from these elements will be -
(A) A_2B (B) AB (C) AB_2 (D) A_2B_3

Q.3 The vapour density of gas A is four times that of B. If molecular mass of B is M, then molecular mass of A is -
(A) M (B) 4M (C) $\frac{M}{4}$ (D) 2M

Q.4 Percentage of copper and oxygen in sample of CuO obtained by different methods were found to be same. This proves the law of -
(A) Constant proportion
(B) Multiple proportion
(C) Reciprocal proportion
(D) None of these

Q.5 6 gm of carbon combines with 32 gm of sulphur to form CS_2. 12 gm of C also combines with 32 gm of oxygen to form carbondioxide. 10 gm of sulphur combines with 10 gm of oxygen to form sulphur dioxide. Which law is illustrated by them -
(A) Law of multiple proportions
(B) Law of constant composition
(C) Law of Reciprocal proportions
(D) Gay Lussac's law

Q.6 Two elements X (at mass 16) and **Y** (at mass 14) combine to form compounds **A, B** and **C**. The ratio of different masses of **Y** which combine with a fixed mass of **X** in A, B and C is 1 : 3 : 5. If 32 parts by mass of **X** combines with 84 parts by mass of **Y** in **B**, then in **C**, 16 parts by mass of **X** will combine with-
(A) 14 parts by mass of **Y**
(B) 42 parts by mass of **Y**
(C) 70 parts by mass of **Y**
(D) 84 parts by mass of **Y**

Q.7 If one mole of ethanol (C_2H_5OH) completely burns to form carbon dioxide and water, the weight of carbon dioxide formed is about –
$C_2H_5OH + 3O_2 \longrightarrow 2CO_2 + 3H_2O$
(A) 22 gm (B) 45 gm
(C) 66 gm (D) 88 gm

Q.8 If LPG cylinder contains mixture of butane and isobutane, then the amount of oxygen that would be required for combustion of 1kg of it will be-
$2C_4H_{10} + 13O_2 \longrightarrow 8CO_2 + 10H_2O$
(A) 1.8 kg (B) 2.7 kg
(C) 4.5 kg (D) 3.58 kg

Q.9 1 gm - atom of nitrogen represents -
(A) 6.02×10^{23} N_2 molecules
(B) 22.4 lit. of N_2 at N.T.P.
(C) 11.2 lit. of N_2 at N.T.P.
(D) 28 gm of nitrogen.

Q.10 The moles of O_2 required for reacting with 6.8 gm of ammonia.
($4NH_3 + 5O_2 \rightarrow 4NO + 6H_2O$) is
(A) 5 (B) 2.5 (C) 1 (D) 0.5

Q.11 If isotopic distribution of C–12 and C–14 is 98% and 2% respectively, then the number of C–14 atoms in 12 gm of carbon is -
(A) 1.032×10^{22} (B) 3.01×10^{22}
(C) 5.88×10^{23} (D) 6.02×10^{23}

Q.12 If 3.01×10^{20} molecules are removed from 98 mg of H_2SO_4, then the number of moles of H_2SO_4 left are -
(A) 0.1×10^{-3} (B) 0.5×10^{-3}
(C) 1.66×10^{-3} (D) 9.95×10^{-2}

Q.13 Total number of atoms of all elements present in 1 mole of ammonium dichromate $[(NH_4)_2Cr_2O_7]$ is

(A) 14 (B) 19

(C) 6×10^{23} (D) 114×10^{23}

Q.14 X gm of Ag was dissolved in HNO_3 and the solution was treated with excess of NaCl. When 2.87 gm of AgCl was precipitated. The value of x is

$Ag + 2HNO_3 \longrightarrow AgNO_3 + NO_2 + H_2O$

$AgNO_3 + NaCl \longrightarrow AgCl + NaNO_3$

(A) 1.08 gm (B) 2.16 gm

(C) 2.70 gm (D) 1.62 gm

Q.15 What mass of calcium chloride in grams would be enough to produce 14.35 gm of AgCl. (At. mass Ca = 40, Ag = 108) –

$CaCl_2 + 2\ AgNO_3 \rightarrow Ca(NO_3)_2 + 2AgCl$

(A) 5.55 gm (B) 8.295 gm

(C) 16.59 gm (D) 11.19 gm

Q.16 Total no. of atoms in 44 gm of CO_2 is -

(A) 6.02×10^{23}

(B) 6.02×10^{24}

(C) 1.806×10^{24}

(D) 18.06×10^{22}

Q.17 If the density of water is 1 gm/cm^3, then the volume occupied by one molecule of water is approximately-

(A) 18 cm^3

(B) 22400 cm^3

(C) 6.02×10^{-23} cm^3

(D) 3.0×10^{-23} cm^3

Q.18 How many grams are contained in 1gm-atom of Na-

(A) 13 gm (B) 23 gm

(C) 1 gm (D) 1/23 gm

Q.19 1.35 gm of pure Ca metal was converted into 1.88 gm of pure CaO. What is atomic weight of Ca -

(A) 40.75 (B) 50

(C) 60 (D) 70

Q.20 The % loss in weight after heating a pure sample of potassium chlorate (M. wt. 122.5) will be –

$2KClO_3 \xrightarrow{\Delta} 2KCl + 3O_2$

(A) 12.25 (B) 24.50

(C) 39.17 (D) 49.00

Q.21 The minimum quantity in gram of H_2S needed to precipitate 63.5 gm of Cu^{+2} will be nearly

$Cu^{+2} + H_2S \longrightarrow CuS + 2H^+$

(A) 63.5 gm (B) 31.75 gm

(C) 34 gm (D) 20 gm

Q.22 Mass of H_2O in 1000 kg $CuSO_4.5H_2O$ is - (Cu = 63.5)

(A) 360.5 kg (B) 36.05 kg

(C) 3605 kg (D) 3.605 kg

Q.23 Phosphine (PH_3) decomposes to produce vapours of phosphorus and H_2 gas. What will be the change in volume when 100 mL of phosphine is decomposed ?

$4PH_3(g) \longrightarrow P_4(g) + 6H_2(g)$

(A) + 50 mL (B) 500 mL

(C) + 75 mL (D) – 500 mL

Q.24 In the reaction $4A + 2B + 3C \rightarrow A_4B_2C_3$, what will be the number of moles of product formed, starting from one mole of A, 0.6 mole of B and 0.72 mole of C ?

(A) 0.25 (B) 0.3

(C) 0.24 (D) 2.32

Q.25 8 gm of O_2 has the same number of molecules as -

(A) 7 gm of CO (B) 14 gm of CO

(C) 14 gm of CO_2 (D) 12 gm of CO_2

Q.26 4.4 gm of CO_2 and 2.24 litre of H_2 at STP are mixed in a container. The total number of molecules present in the container will be -

(A) 6.022×10^{23}

(B) 1.2044×10^{23}

(C) 2 moles

(D) 6.023×10^{24}

Q.27 Find the volume of CO_2 obtained at S.T.P. on heating 200 gm of 50% pure $CaCO_3$ -
(A) 11.2 litre (B) 22.4 litre
(C) 44.8 litre (D) None of these

Q.28 A gas has a vapour density 11.2. The volume occupied by 1 g of the gas at NTP is :
(A) 1 L (B) 11.2 L
(C) 22.4 L (D) 4 L

Q.29 A sample of AlF_3 contains 3.0×10^{24} F^- ions. The number of formula units in this sample are -
(A) 9.0×10^{24} (B) 3.0×10^{24}
(C) 0.75×10^{24} (D) 1.0×10^{24}

Q.30 One mole of nitrogen gas is the volume of
(A) 1 litre of nitrogen at S.T.P.
(B) 32 litre of nitrogen at S.T.P.
(C) 22.4 litre of nitrogen at S.T.P.
(D) 11.2 litre of nitrogen at S.T.P.

Q.31 The mass of $CaCO_3$ produced when carbon dioxide is passed in excess through 500 ml of 0.5 M $Ca(OH)_2$ will be-
(A) 10 gm (B) 20 gm
(C) 50 gm (D) 25 gm.

Q.32 Atomic weight of Ne is 20.2 Ne is mixture of Ne^{20} and Ne^{22}, relative abundance of heavier isotope is :
(A) 90 (B) 20 (C) 40 (D) 10

Q.33 Which of the following contains maximum number of molecules -
(A) 400 cc of O_2 at S.T.P.
(B) 500 cc of NH_3 at S.T.P.
(C) 250 cc of SO_2 at S.T.P.
(D) 150 cc of CO_2 at S.T.P.

Q.34 1.6 g of sulphur was burnt in the air to form SO_2. The number of molecules of SO_2 introduced into the air will be -
(A) 6.02×10^{23} (B) 3.01×10^{23}
(C) 6.02×10^{22} (D) 3.01×10^{22}

Q.35 1.5 gm of divalent metal displaced 4 gm of copper (at. wt. = 64) from a solution of copper sulphate. The atomic weight of the metal is-
(A) 12 (B) 24
(C) 48 (D) 6

Q.36 Avogadro's number of Rupees can be spent in .....years if 10 lac rupees per second are spent
(A) 1.91×10^{10} year (B) 2.91×10^{10} year
(C) 3.91×10^{10} year (D) 4.91×10^{10} year

EXERCISE-2

Q.1 4.0 g of caustic soda (mol mass 40) contains same number of sodium ions as are present in-
(A) 10.6 g of Na_2CO_3 (mol. mass 106)
(B) 58.5 g of NaCl (Formula mass 58.5)
(C) 100 ml of 0.5 M Na_2SO_4 (Formula mass 142)
(D) 1mol of $NaNO_3$ (mol. mass 85)

Q.2 0.01 mole of iodoform (CHI_3) reacts with Ag to produce a gas whose volume at NTP is -
(A) 224 ml
(B) 112 ml
(C) 336 ml
(D) None of these

Q.3 If 1.6 gms of SO_2 1.5×10^{22} molecules of H_2S are mixed and allowed to remain in contact in a closed vessel until the reaction
$2H_2S + SO_2 \longrightarrow 3S + 2H_2O$,
proceeds to completion. Which of the following statement is true ?
(A) Only 'S' and 'H_2O' remain in the reaction vessel
(B) 'H_2S' will remain in excess
(C) 'SO_2' will remain in excess
(D) None of these

Q.4 1.0 gm of a metal combines with 8.89 gms of Bromine. Equivalent weight of metal is nearly : (at. wt. of Br = 80)
(A) 8 (B) 9 (C) 10 (D) 7

Q.5 2.8 gm of iron displaces 3.2 gm of copper from a solution of copper sulphate solution. If the equivalent mass of iron is 28, then equivalent mass of copper will be -
(A) 16 (B) 32 (C) 48 (D) 64

Q.6 2.76 gm of silver carbonate on being strongly heated yields a residue weighing -
(A) 2.16 gm (B) 2.48 gm
(C) 2.32 gm (D) 2.64 gm

Q.7 A hydrocarbon contains 80% of carbon, then the hydrocarbon is -
(A) CH_4 (B) C_2H_4
(C) C_2H_6 (D) C_2H_2

Q.8 A giant molecule contains 0.25% of a metal whose atomic weight is 59. Its molecule contains one atom of that metal. Its minimum molecular weight is -
(A) 5900 (B) 23600
(C) 11800 (D) $\frac{100 \times 59}{0.4}$

Q.9 $CaCO_3$ is 90% pure. Volume of CO_2 collected STP when 10 gms of $CaCO_3$ is decomposed is -
(A) 2.016 litres (B) 1.008 litres
(C) 10.08 litres (D) 20.16 litres

Q.10 The formula of a metal oxide is Z_2O_3. If 6 mg. of hydrogen is required for complete reduction of 0.1596 gm metal oxide, then the atomic weight of metal is -
(A) 227.9 (B) 159.6
(C) 79.8 (D) 55.8

Q.11 Percentage of Se in peroxidase anhydrous enzyme is 0.5% by weight (at. wt. = 78.4) then min. mol. wt. of peroxidase anhydrous enzymes is -
(A) 1.568×10^4 (B) 1.568×10^3
(C) 15.68 (D) 2.136×10^4

Q.12 The mass of carbon anode consumed (giving only carbondioxide) in the production of 270 Kg of aluminium metal from bauxite by the Hall process is -
(A) 180 Kg (B) 270 Kg
(C) 240 Kg (D) 90 Kg

Q.13 How many moles of lead (II) chloride will be formed from a reaction between 6.5 g of PbO and 3.2 g of HCl ?
(Atomic wt. of Pb = 207) -
(A) 0.011 (B) 0.029
(C) 0.044 (D) 0.333

Q.14 The emperical formula of an organic compound is CH_2. One mole of this compound has a mass 42 gm. Its molecular formula is -
(A) CH_2
(B) C_3H_6
(C) C_2H_2
(D) C_3H_8

Q.15 The mass of 70% pure H_2SO_4 required for neutralisation of 1 mol of NaOH -
(A) 49 gm
(B) 98 gm
(C) 70 gm
(D) 34.3 gm

Passage :

10 moles of SO_2 and 4 moles of O_2 are mixed in a closed vessel of volume 2 litres. The mixture is heated in presence of Pt catalyst. Following reaction takes place :
$2SO_2(g) + O_2(g) \longrightarrow 2SO_3(g)$
Assuming the reaction proceeds to completion.

Q.16 Select the correct statement -
(A) SO_2 is the limiting reagent
(B) O_2 is the limiting reagent
(C) both SO_2 and O_2 are limiting reagent
(D) cannot be predicted

Q.17 Number of moles of SO_3 formed in the reaction will be -
(A) 10 (B) 4
(C) 8 (D) 14

Q.18 Number of moles of excess reactant remaining -
(A) 4 (B) 2
(C) 6 (D) 8

Q.19 The calculation on the given reaction is based on -
(A) Boyle's law
(B) Charle's law
(C) Gay-Lussac's law
(D) Avogadro's hypothesis

Q.20 Total number of moles of gaseous component will ..... after the reaction -
(A) increase
(B) decrease
(C) remain same
(D) may increase or decrease

Q.21 Calculate the gm quantity of Na_2CO_3 which has same No. of atoms as the No. of protons present in 10 gm $CaCO_3$ -
(A) 20 gm (B) 88.33 gm
(C) 44 gm (D) 60 gm

Q.22 A sample of hard water is found to contain 40 mg of Ca^{+2} ion per litre. The amount of washing soda (Na_2CO_3) required to soften five litres of the sample would be -
(A) 1.06 gm (B) 5.3 gm
(C) 53 mg (D) 530 mg

Q.23 The mass of oxygen that would be required to produce enough CO, which completely reduces 1.6 kg Fe_2O_3 (at. mass Fe = 56) is -
(A) 240 gm (B) 480 gm
(C) 720 gm (D) 960 gm

Q.24 The amount of sulphur required to produce
100 moles of H_2SO_4 is -
(A) 3.2×10^3 gm (B) 32.65 gm
(C) 32 gm (D) 3.2 gm

Q.25 The vapour density of a mixture containing NO_2 and N_2O_4 is 38.3 at 27° C. The moles of NO_2 in 100 mole mixture is -
(A) 33.48 (B) 53.52
(C) 28.3 (D) 76.6

Q.26 Assuming that petrol is iso-octane (C_8H_{18}) and has density 0.8 gm/ml, 1.425 litre of petrol on complete combustion will consume oxygen -
(A) 50 L
(B) 125 L
(C) 125 mol
(D) 50 mol

Q.27 The conversion of oxygen to ozone occurs to the extent of 15% only. The mass of ozone that can be prepared from 67.2 L of oxygen at S.T.P. will be -
(A) 14.4 gm
(B) 96 gm
(C) 640 gm
(D) 64 gm

Q.28 The density of a liquid is 1.2 g/ml. There are 35 drops in 2 mL. The number of molecules in 1 drop is :
(molecular weight of liquid = 70)
(A) $\frac{12}{35}$ N_A (B) $\left(\frac{1}{35}\right)^2 N_A$
(C) $\frac{1.2}{(35)^2} N_A$ (D) 12 N_A

Q.29 Number of electrons present in 3.6 mg of NH_4^+ are -
(A) 1.2×10^{21}
(B) 1.2×10^{20}
(C) 1.2×10^{22}
(D) 2×10^{-3}

Q.30 If N_A is the Avogadro's number then number of valence electrons in 4.8 g of O^{2-} is -
(A) 2.4 N_A
(B) 4.2 N_A
(C) 1.6 N_A
(D) 3.2 N_A

Q.31 Which of the following has greatest number of atoms ?
(A) 1 g of butane (C_4H_{10})
(B) 1 g of nitrogen (N_2)
(C) 1 g of silver (Ag)
(D) 1 g of water (H_2O)

Q.32 The molar ratio of Fe^{2+} to Fe^{3+} in a mixture of $FeSO_4$ and $Fe_2(SO_4)_3$ having equal number of sulphate ions in both ferrous and ferric sulphates is -
(A) 1 : 2
(B) 3 : 2
(C) 2 : 3
(D) none of these

Q.33 Study the following table :

Compound	**Mass of the compound (in gram) taken**
I. CO_2	4.4
II. NO_2	2.3
III. H_2O_2	6.8
IV. SO_2	3.2

Which two compounds have least mass of oxygen ?
(A) II and IV (B) I and III
(C) I and II (D) III and IV

Q.34 How many electrons are present in 2×10^{-3} mol of $^{18}_{8}O^{-2}$?
(A) 1.2×10^{21} (B) 9.6×10^{21}
(C) 1.2×10^{22} (D) 1.9×10^{22}

Q.35 What is the empirical formula of vanadium oxide if 2.74 g of metal oxide contains 1.53 g of metal ? (V = 52)
(A) V_2O_3 (B) VO
(C) V_2O_5 (D) V_2O_7

Q.36 What is the formula of a substance with mass percentages of 35.79 % for S, 62.92 % for O and 1.13 % for H ?
(A) H_2SO_3 (B) H_2SO_4
(C) $H_2S_2O_7$ (D) $H_2S_2O_8$

Q.37 The chloride of a metal (M) contains 65.5 % of chlorine. 100 ml of the vapour of the chloride of the metal at STP weight 0.72 g. The molecular formula of the metal chloride is -
(A) MCl_3 (B) MCl (C) MCl_2 (D) MCl_4

Q.38 0.44 g of a colourless oxide of nitrogen occupies 224 ml at STP. The compound is -
(A) N_2O (B) NO
(C) N_2O_2 (D) NO_2

Q.39 How many grams of copper will be replaced in 2L of a 1.50 M $CuSO_4$ solution if the later is made to react with 27.0 gm of aluminium ?
[Cu = 63.5, Al = 27.0]
(A) 190.50 g (B) 95.25 g
(C) 31.75 g (D) 10.65 g

Q.40 22.4 litres of H_2S and 22.4 litres of SO_2 both at STP are mixed together. The amount of sulphur precipitated as a result of chemical calculation is :
($2H_2S + SO_2 \rightarrow 3S + 2H_2O$)
(A) 16 g (B) 23 g
(C) 48 g (D) 96 g

Q.41 Caffeine has a molecular weight of 194. If it contains 29 % by mass of nitrogen, number of atoms of nitrogen in one molecule of caffeine is-
(A) 4 (B) 6
(C) 2 (D) 3

Q.42 Sulphur trioxide is prepared by the following two reactions.
$S_8(s) + 8O_2(g) \rightarrow 8SO_2(g)$
$2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$
How many grams of SO_3 are produced from 1 mol of S_8?
(A) 1280.0 (B) 640.0
(C) 960.0 (D) 320.0

Q.43 An organic compound contains 20 atoms of carbon per molecule, and contains 70% carbon by mass. The molecular mass of the organic compound is approximately.
(A) 465.0 (B) 342.85
(C) 415.0 (D) 667.0

EXERCISE-3

Old Examination Questions [AIEEE/JEE Main]

Q.1 The weight of 2.01×10^{23} molecules of CO is- **[AIEEE-2002]**

(A) 9.3 gm (B) 7.2 gm
(C) 1.2 gm (D) 3 gm

Q.2 In an organic compound of molar mass 108 gm mol^{-1} C, H and N atoms are present in 9 : 1 : 3.5 by weight. Molecular formula can be – **[AIEEE-2002]**

(A) $C_6H_8N_2$ (B) $C_7H_{10}N$
(C) $C_5H_6N_3$ (D) $C_4H_{18}N_3$

Q.3 Number of atoms in 560 gm of Fe (atomic mass 56 g mol^{-1}) is – **[AIEEE-2003]**

(A) is twice that of 70 gm N
(B) is half that of 20 gm H
(C) both are correct
(D) None is correct

Q.4 6.02×10^{20} molecules of urea are present in 100 ml of its solution. The concentration of urea solution is - **[AIEEE-2004]**

(A) 0.001 M (B) 0.01 M
(C) 0.02 M (D) 0.1 M

(Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1})

Q.5 How many moles of magnesium phosphate, $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms ? **[AIEEE 2006]**

(A) 3.125×10^{-2} (B) 1.25×10^{-2}
(C) 2.5×10^{-2} (D) 0.02

Q.6 In the reaction, **[AIEEE 2007]**

$$2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2Al^{3+}_{(aq)} + 6Cl^{-}_{(aq)} + 3H_2(g),$$

(A) 6L $HCl_{(aq)}$ is consumed for every 3L $H_2(g)$ produced
(B) 33.6 L $H_{2(g)}$ is produced regardless of temperature and pressure for every mole Al that reacts
(C) 67.2 L $H_{2(g)}$ at STP is produced for every mole Al that reacts
(D) 11.2 L $H_{2(g)}$ at STP is produced for every mole $HCl_{(aq)}$ consumed

Q.7 The concentrated sulphuric acid that is labelled commercially is 95% H_2SO_4 by weight. If the density of this commercial acid is 1.834 g cm^{-3} , the molarity of this solution is : **[JEE Main Online– 2012]**

(A) 17.8 M (B) 15.7 M
(C) 10.5 M (D) 12.0 M

Q.8 The ratio of number of oxygen atoms (O) in 16.0 g ozone (O_3), 28.0 g carbon monoxide (CO) and 16.0 g oxygen (O_2) is (Atomic mass : C = 12, O = 16 and Avogadro's constant $N_A = 6.0 \times 10^{23}$ mol^{-1}) : **[JEE Main Online– 2012]**

(A) 3 : 1 : 1
(B) 1 : 1 : 2
(C) 3 : 1 : 2
(D) 1 : 1 : 1

Q.9 When $CO_2(g)$ is passed over red hot coke it partially gets reduced to CO(g). Upon passing 0.5 litre of $CO_2(g)$ over red hot coke, the total volume of the gases increased to 700 mL . The composition of the gaseous mixture at STP is : **[JEE Main Online– 2012]**

(A) CO_2 = 200 mL ; CO = 500 mL
(B) CO_2 = 350 mL ; CO = 350 mL
(C) CO_2 = 0.0 mL ; CO = 700 mL
(D) CO_2 = 300 mL ; CO = 400 mL

Q.10 The ppm level of F^- in a 500g sample of a tooth paste containing 0.2g F^- is : **[JEE Main Online– 2012]**

(A) 250 (B) 200
(C) 400 (D) 1000

Q.11 A gaseous hydrocarbon gives upon combustion 0.72 g. of water and 3.08 g. of CO_2. The empirical formula of the hydrocarbon is – **[JEE Main 2013]**

(A) C_6H_5
(B) C_7H_8
(C) C_2H_4
(D) C_3H_4

Q.12 Number of atoms in the following samples of substances is the largest in :

[JEE Main Online– 2013]

(A) 4.0g of hydrogen (H_2)
(B) 71.0g of chlorine (Cl_2)
(C) 127.0g of iodine (I_2)
(D) 48.0g of magnesium (Mg)

Q.13 6 litres of an alkene require 27 litres of oxygen at constant temperature and pressure for complete combustion. The alkene is – **[JEE Main Online– 2013]**

(A) Ethene (B) Propene
(C) 1 – Butene (D) 2–Butene

Q.14 Dissolving 120 g of a compound of (mol. wt. 60) in 1000 g of water gave a solution of density 1.12 g/mL. The molarity of the solution is : **[JEE Main Online– 2014]**

(A) 1.00 M (B) 2.00 M
(C) 2.50 M (D) 4.00 M

Q.15 The amount of oxygen in 3.6 moles of water is **[JEE Main Online– 2014]**

(A) 115.2 g (B) 57.6 g
(C) 28. 8 g (D)18.4 g

Q.16 A gaseous compound of nitrogen and hydrogen contains 12.5% (by mass) of hydrogen. The density of the compound relative to hydrogen is 16. The molecular formula of the compound is:

[JEE Main Online– 2014]

(A) NH_2 (B) N_3H
(C) NH_3 (D) N_2H_4

Q.17 The amount of $BaSO_4$ formed upon mixing 100 mL. of 20.8% $BaCl_2$ solution with 50 mL of 9.8% H_2SO_4 solution will be : (Ba = 137, Cl = 35.5, S = 32, H = 1 and O = 16)

[JEE Main Online– 2014]

(A) 23.3 g (B) 11.65 g
(C) 30.6 g (D) 33.2 g

Q.18 The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. Wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin ?

[JEE Main 2015]

(A) $\frac{1}{103}$ (B) $\frac{1}{206}$
(C) $\frac{2}{309}$ (D) $\frac{1}{412}$

Q.19 At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for combustion. After combustion the gaseous occupy 345 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is :

[JEE Main 2016]

(A) C_3H_6 (B) C_3H_8
(C) C_4H_8 (D) C_4H_{10}

Q.20 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is –

[JEE Main 2017]

(A) 118.6 (B) 11.86
(C) 1186 (D) 84.3

Q.21 The most abundant elements by mass in the body of a healthy human adult are : Oxygen (61.4%), Carbon (22.9 %), Hydrogen (10.0%) and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is

[JEE Main 2017]

(A) 7.5 kg (B) 10 kg
(C) 15 kg (D) 37.5 kg

Q.22 Excess of NaOH (aq) was added to 100 mL of $FeCl_3$ (aq) resulting into 2.14 g of $Fe(OH)_3$. The molarity of $FeCl_3$ (aq) is

(Given molar mass of Fe = 56 g mol^{-1} and molar mass of Cl = 35.5 g mol^{-1})

[JEE Main Online– 2017]

(A) 0.3 M (B) 0.2 M
(C) 0.6 M (D) 1.8 M

Q.23 The ratio of mass percent of C and H of an organic compound ($C_xH_yO_z$) is 6 : 1. If one molecule of the above compound ($C_xH_yO_z$) contains half as much oxygen as required to burn one molecule of compound C_xH_y completely to CO_2 and H_2O. The empirical formula of compound $C_xH_yO_z$ is -

[JEE Main - 2018]

(A) $C_3H_6O_3$ (B) C_2H_4O

(C) $C_3H_4O_2$ (D) $C_2H_4O_3$

Q.24 The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.Ca(OH)_2]$ to :

[JEE Main - 2018]

(A) $[CaF_2]$

(B) $[3(CaF_2).Ca(OH)_2]$

(C) $[3Ca_3(PO_4)_2.CaF_2]$

(D) $[3\{Ca(OH)_2\}.CaF_2]$

Q.25 Biochemical oxygen demand (BOD) value can be a measure of water pollution caused by the organic matter. Which of the following statements is correct ?

[JEE-Main Online-2018]

(A) Polluted water has BOD value higher than 10 ppm

(B) Aerobic bacteria decreases the BOD value

(C) Anaerobic bacteria increases the BOD value

(D) Clean water has BOD value higher than 10 ppm

EXERCISE-4

Old Examination Questions [IIT JEE Advanced]

Q.1 The weight of 1×10^{22} molecules of $CuSO_4.5H_2O$ is - **[IIT-1991]**

(A) 41.59 g (B) 415.9 g
(C) 4.159 g (D) None of these

Q.2 Rearrange the following (I to IV) in the order of increasing masses and choose the correct Answer from (A), (B), (C) and (D)
(At. mass : N = 14, O = 16, Cu = 63)
(I) 1 molecule of O_2 **[IIT-1993]**
(II) 1 atom of nitrogen
(III) 1×10^{-10} (g molecular mass of oxygen)
(IV) 1×10^{-7} (g atomic mass of copper
(A) II < I < III < IV (B) IV < III < II < I
(C) II < III < I < IV (D) III < IV < I < II

Q.3 One mole of calcium phosphide on reaction with excess of water gives **[IIT-1999]**

(A) One mole of phosphine
(B) Two moles of Phosphoric acid
(C) Two moles of phosphine
(D) One mole of phosphorus pentoxide

Q.4 At 100°C and 1 atm, if the density of liquid water is 1.0 g cm^{-3} and that of water vapour is 0.0006 g cm^{-3} , then the volume occupied by water molecules in 1 litre of steam at that temperature is **[IIT-2000]**

(A) 6 cm^3 (B) 60 cm^3
(C) 0.6 cm^3 (D) 0.06 cm^3

Q.5 How many moles of electron weigh one kilogram ? **[IIT- 2002]**

(A) 6.023×10^{23}
(B) $\frac{1}{9.108} \times 10^{31}$
(C) $\frac{6.023}{9.108} \times 10^{54}$
(D) $\frac{1}{9.108 \times 6.023} \times 10^{8}$

Q.6 In which of the following number of atoms are maximum ? **(IIT-2003)**

(A) 24 gms C
(B) 27 gms of Al
(C) 56 gms of Fe
(D) 108 gms of Ag

Q.7 Give that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5% 90% and 5% respectively, the atomic mass of Fe is - **[IIT-2009]**

(A) 55.85 (B) 55.95
(C) 55.75 (D) 56.05

Q.8 The simplest formula of a compound containing 50% of element X (At. mass = 10) and 50% of the element Y (At. mass = 20) is : **[IIT-2011]**

(A) XY (B) X_2Y
(C) XY_2 (D) X_2Y_3

Q.9 The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to - **[IIT-2011]**

Q.10 If the value of Avogadro number is 6.023×10^{23} mol^{-1} and the value of Boltzmann constant is 1.380×10^{-23} J K^{-1}, then the number of significant digits in the calculated value of the universal gas constant is **[JEE Advance – 2014]**

Q.11 The ammonia prepared by treating ammonium sulphate with calcium hydroxide is completely used by $NiCl_2.6H_2O$ to form a stable coordination compound. Assume that both the reactions are 100% complete. If 1584 g of ammonium sulphate and 952 g of $NiCl_2.6H_2O$ are used in the preparation, the combined weight (in grams) of gypsum and the nickel-ammonia coordination compound thus produced is(Atornic weights in g mol^{-1}: H = 1, N = 14, O = l6, S = 32, Cl=35.5, Ca = 40, Ni = 59) **[JEE Advance – 2018]**

ANSWER KEYS

EXERCISE-1

1.(C)	**2.**(C)	**3.**(B)	**4.**(A)	**5.**(C)	**6.**(C)	**7.**(D)
8.(D)	**9.**(C)	**10.**(D)	**11.**(A)	**12.**(B)	**13.**(D)	**14.**(B)
15.(A)	**16.**(C)	**17.**(D)	**18.**(B)	**19.**(A)	**20.**(C)	**21.**(C)
22.(A)	**23.**(C)	**24.**(C)	**25.**(A)	**26.**(B)	**27.**(B)	**28.**(A)
29.(D)	**30.**(C)	**31.**(D)	**32.**(D)	**33.**(B)	**34.**(D)	**35.**(B)
36.(A)						

EXERCISE-2

1.(C)	**2.**(B)	**3.**(C)	**4.**(B)	**5.**(B)	**6.**(A)	**7.**(C)
8.(B)	**9.**(A)	**10.**(D)	**11.**(A)	**12.**(D)	**13.**(B)	**14.**(B)
15.(C)	**16.**(B)	**17.**(C)	**18.**(B)	**19.**(C)	**20.**(B)	**21.**(B)
22.(D)	**23.**(B)	**24.**(A)	**25.**(A)	**26.**(C)	**27.**(A)	**28.**(C)
29.(A)	**30.**(A)	**31.**(A)	**32.**(B)	**33.**(A)	**34.**(C)	**35.**(C)
36.(C)	**37.**(A)	**38.**(A)	**39.**(B)	**40.**(C)	**41.**(A)	**42.**(B)
43.(B)						

EXERCISE-3

1.(A)	**2.**(A)	**3.**(C)	**4.**(B)	**5.**(A)	**6.**(D)	**7.**(A)
8.(D)	**9.**(D)	**10.**(C)	**11.**(B)	**12.**(A)	**13.**(B)	**14.**(B)
15.(B)	**16.**(D)	**17.**(B)	**18.**(D)	**19.**(B)	**20.**(D)	**21.**(A)
22.(B)	**23.** (D)	**24.** (C)	**25.** (A)			

EXERCISE-4

1.(C)	**2.**(A)	**3.**(C)	**4.**(C)	**5.**(D)	**6.**(A)	**7.**(B)
8.(B)	**9.** 6	**10.** 4	**11.** [2992.00]			

All exercise's detailed solutions are available at www.careerpoint.ac.in

EXERCISE-4

1.[C] 6.023×10^{23} molecule = 249.5

1×10^{22} ——— $\frac{249.5}{6.023\times10^{23}} \times 1 \times 10^{22}$

= 4.159 gm

2.[A] (I) Mass of one molecule of O_2 = 32 amu
$= 32 \times 1.66 \times 10^{-24}$ gm

(II) Mass of one atom of Nitrogen = 28 amu $= 28 \times 1.66 \times 10^{-24}$ gm

(III) 1×10^{-10} (gm molecular mass of oxygen) $= 1 \times 10^{-10} \times 32$ gm

(iv) 1×10^{-7} (gm atomic mass of copper) $= 1 \times 10^{-7} \times 63.5$ gm

Correct order of increasing masses
= II < I < III < IV

3.[C] $Ca_3P_2 + 6H_2O \rightarrow 3Ca(OH)_2 + 2PH_3$

4.[C] $\text{mass}_{H_2O_{(\ell)}} = \text{mass}_{H_2O_{(v)}}$

$(V \times d)_{H_2O_{(\ell)}} = (V \times d)_{H_2O_{(V)}}$

$V \times 1 = 1000 \times 0.0006$

$V_{H_2O_{(\ell)}} = 0.6$ cc

5.[D] Mass of an electron = 9.108×10^{-31} kg
Mass of one mole of electron
$= 9.108 \times 10^{-31} \times 6.023 \times 10^{23}$ Kg
$= 9.108 \times 10^{-31} \times 6.023 \times 10^{23}$ Kg contains
= 1 mole of electron

1 kg contains = $\frac{1}{9.108\times10^{-8}\times6.023}$

$= \frac{1}{9.108\times6.023} \times 10^{8}$ kg

6.[A] No. of atom = $\frac{\text{mass}}{\text{At.mass}} \times N_A$

7.[B] $\frac{54\times5+56\times40+57\times5}{100} = 55.95$

8.[B]

X	$\frac{50}{10}$	2
X	$\frac{50}{20}$	1

E.F. $\Rightarrow X_2Y$

9. $N_1V_1 = N_2V_2$
$0.1\ V = 30 \times 0.02$
v = 6 ml

10. R = (Avogadro number) × (Boltzmann constant)
Both contains four significant digits
$\therefore$ answer must contains four significant digits

11. [2992.00]

PERMUTATIONS & COMBINATIONS

JEE MAIN SYLLABUS

1. *Factorial Notation*
2. *Fundamental Principles of Operation*
3. *Combinations*
4. *Permutations*
5. *Division into Groups*
6. *Permutations in which the operation of Selection is necessary*
7. *Dearrangement Theorem*
8. *Multinomial Theorem & Its Applications*
9. *Divisibility of Numbers*
10. *Sum of Numbers*

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

A. Write Question Number (QN) which you are unable to solve at your own in **column A**.

B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.

C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Exercise-1		
Exercise-2		
Exercise-3		
Exercise-4		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Step-3: Solve Questions of Column B

Step-4: Solve questions from other Question Bank, Problem book etc.

Permutations & Combinations

KEY CONCEPT

1. Fundamental Principles of Operation

When one or more operations can be accomplished by number of ways then there are two principles to find the total number of ways to accomplish one, two, or all of the operations without counting them as follows:

1.1 Fundamental Principle of Multiplication :

Let there are two parts A and B of an operation and if these two parts can be performed in m and n different number of ways respectively, then that operation can be completed in m × n ways.

1.2 Fundamental Principle of addition :

If there are two operations such that they can be done independently in m and n ways respectively, then any one of these two operations can be done by (m + n) number of ways.

2. Combinations

The different groups or selections of a given number of things by taking some or all at a time without paying any regard to their order, are called their **combinations.**

The number of combinations of n **different** things taken r at a time is denoted by

$${}^nC_r \text{ or } C(n, r)$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

So $${}^nC_r = \frac{n(n-1)(n-2).....(n-r+1)}{r!}$$

Particular cases : $${}^nC_r = \frac{{}^nP_r}{r!}$$

$${}^nC_n = 1$$

$${}^nC_0 = 1$$

Some Important Results :

* $${}^nC_r = {}^nC_{n-r}$$
* $${}^nC_x = {}^nC_y \Rightarrow x + y = n$$
* $${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$
* $${}^nC_r = \frac{n}{r}.{}^{n-1}C_{r-1}$$
* $${}^nC_r = \frac{1}{r}(n-r+1)\,{}^nC_{r-1}$$
* $${}^nC_1 = {}^nC_{n-1} = n$$
* Greatest value of ${}^nC_r$
  = ${}^nC_{n/2}$, when n is even
  = ${}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$, when n is odd

2.1 Restricted Combinations :

The number of combinations of n different things taking r at a time

(i) When p particular things are always to be included = $\mathbf{{}^{n-p}C_{r-p}}$

(ii) When p particular things are always to be excluded = $\mathbf{{}^{n-p}C_r}$

(iii) When p particular things are always included and q particular things are always excluded
= $\mathbf{{}^{n-p-q}C_{r-p}}$

2.2 Total number of combinations in different cases :

(i) The number of combinations of n **different things** taking some or all (or atleast one) at a time = ${}^nC_1 + {}^nC_2 + ...+ {}^nC_n$
= $\mathbf{2^n - 1}$

(ii) The number of ways to select some or all out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is = **(p +1) (q +1) (r +1) – 1**

(iii) The number of ways to select some or all out of (p + q + t) things where p are alike of first kind, q are alike of second kind and remaining t are different is = **(p +1) (q +1) 2^t – 1**

3. Permutations

An arrangement of some given things taking some or all of them, is called a **permutation** of these things.

For Example, three different things a, b and c are given, then different arrangements which can be made by taking two things from the three given things are

ab, ac, bc, ba, ca, cb

Therefore, the number of permutations will be 6.

3.1 The number of permutations of **n <u>different</u>** things taken r at a time is ${}^nP_r$, where

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

$$ = n\,(n-1)\,(n-2)\,.....\,(n-r+1) $$

The number of permutations of n dissimilar things taken all at a time = ${}^nP_n = n!$

Difference between Permutation and Combination :

(i) In combinations the order of things has no role to play, while in permutations the order of things plays an important role.

(ii) In combinations the different groups are formed by taking some or all given things, while in case of permutations all possible arrangements of things are made in each group. Thus the number of permutations is always greater than the number of combinations.

Note:

- In general, we use the methods of finding combinations in case of forming groups, teams or committees, while the methods of finding permutations are used in forming numbers with the help of digits, words with the help of letters, distribution of prizes etc.

3.2 Permutations in which all things are not different :

The number of permutations of n things taken **<u>all at a time</u>** when p of them are alike and of one kind, q of them are alike and of second kind, r of them are alike and of third kind and all remaining being different is

$$ \frac{n!}{p!q!r!} $$

3.3 Permutations in which things may be repeated :

The number of permutations of n **<u>different things</u>** taken r at a time when each thing can be used once, twice, ...upto r times in any permutation is $\mathbf{n^r}$.

In particular, in above case when n things are taken at a time then total number of permutation is $\mathbf{n^n}$.

3.4 Restricted Permutations : If in a permutation, some particular things are to be placed at some particular places or some particular things are always to be included or excluded, then it is called a **restricted permutation.** The following are some of the restricted permutations.

(i) The number of permutations of n dissimilar things taken r at a time, when m particular things always occupy definite places = $\mathbf{{}^{n-m}P_{r-m}}$

(ii) The number of permutations of n different things taken altogether when r particular things are to be placed at some **r** given places.

$$ = {}^{n-r}\mathbf{P_{n-r} = (n-r)\,!} $$

(iii) The number of permutations of n different things taken r at a time, when m particular things are always to be excluded = $\mathbf{{}^{n-m}P_r}$

(iv) The number of permutations of n different things taken r at a time when m particular things are always to be included

$$ = {}^{n-m}\mathbf{C_{r-m} \times r!} $$

3.5 Permutation of numbers when given digits include zero :

If the given digits include 0, then two or more digit numbers formed with these digits cannot have 0 on the extreme left. In such cases we find the number of permutations in the following two ways.

(i) (The number of digits which may be used at the extreme left) x (The number of ways in which the remaining places may be filled up)

(ii) If given digits be n (including 0) then total number of m- digit numbers formed with them

$$ = {}^nP_m - {}^{n-1}P_{m-1}. $$

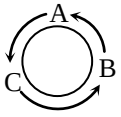
because ${}^{n-1}P_{m-1}$ is the number of such numbers which contain 0 at extreme left.

3.6 Circular Permutations

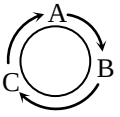
Till now we have calculated the number of linear permutation in which things are arranged in a row. Now we shall find the number of permutations in which things are arranged in a circular shape. Such permutations are named as **circular permutations**. Thus an arrangement of some given things round a circle is called their circular permutation.

It should be noted that in a circular permutation initial and final position of things can not be specified. Thus all linear permutations of some given things having the same order of elements will give the same circular permutation.

For example, there are 6 linear permutations of three letters A, B and C taken all at a time. These are ABC, ACB, BAC, BCA, CAB, CBA



(Anti- clockwise order)



(Clockwise order)

Since the arrangements ABC, BCA, CAB, are in the same order (clockwise order), therefore these three linear permutations are equal to one circular permutation.

From this example, it is clear that from a circular permutation of three things, there correspond three linear permutations. Thus, we conclude that if x be the number of circular permutations of 3 given things then the number of their linear permutations will be 3x.so

$$3x = 3! \Rightarrow x = \frac{3!}{3}.$$

In a similar way it can be seen that if x be the number of circular permutations of n different things taking r at a time, then

$$rx = {}^{n}P_r \Rightarrow x = {}^{n}P_r / r$$

Thus, we obtain the following results for the number of circular permutations.

3.6.1 Number of Circular Permutations :

(i) The number of Circular permutations of n **different** things taking r at a time $\frac{{}^{n}P_r}{r}$, when clockwise and anti-clockwise orders are treated as different.

(ii) The number of circular permutations of n **different** things taking altogether $\frac{{}^{n}P_n}{n}$, when clockwise and anti clockwise orders are treated as different.

(iii) The number of Circular permutations of n **different** things taking r at a time $\frac{{}^{n}P_r}{2r}$, when the above two orders are treated as same.

(iv) The number of circular permutations of n **different** things taking altogether $\frac{{}^{n}P_n}{2n} = \frac{1}{2}(n-1)!$, when above two orders are treated as same.

Note :

- In a circular permutation the relative position among the things is important whereas the place of a thing has no significance. Thus in a circular permutation first thing can be placed anywhere. This operation can be done only in one way, then relative order begins. Thus the ways for performing remaining parts of the operation can be calculated just like the calculation of linear permutation for example to place 8 different things round a circle, first we place any one thing at any one place, there will be only one way to perform this operation. Then remaining 7 things will occupy remaining 7 places, for which the numbers of ways = 7! . Thus required number of circular permutations is 7!
- In a garland of flowers or a necklace of beads, it is difficult to distinguish clockwise and anticlockwise orders of things, so a circular permutation under both these orders is considered to be the same.

3.6.2 Restricted Circular Permutations :

When there is a restriction in a Circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

4. Division into Groups

(i) The number of ways in which (p + q) things can be divided into two groups of p and q things is

$${}^{p+q}C_p = {}^{p+q}C_q = \frac{(p+q)!}{p!\,q!}$$

Particular case : when p = q, then total number of combinations are

(a) $\frac{2p!}{(p!)^2}$ when groups are differentiable.

(b) $\frac{2p!}{2!\,(p!)^2}$ when groups are not differentiable.

(ii) The number of ways in which (p + q + r) things can be divided into three groups containing p, q and r things is

$$\frac{(p+q+r)!}{p!\,q!\,r!}$$

Particular case :

when p = q = r, then total number of combinations are

(a) $\frac{3p!}{(p!)^3}$ when groups are differentiable.

(b) $\frac{3p!}{3!\,(p!)^3}$ when groups are not differentiable.

5. Permutations in Which the Operation of Selection is Necessary

There are questions of permutation in which we have to start with the operation of selection for the given number of things. After this we calculate the number of different arrangements for each of such selected group.

6. Dearrangement Theorem

Any change in the given order of the things is called a **Dearrangement** .

(i) If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is $n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....(-1)^n\frac{1}{n!}\right]$

(ii) If n things are arranged at n places then the number of ways to rearrange exactly r things at right places is

$$\frac{n!}{r!}\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....(-1)^{n-r}\frac{1}{(n-r)!}\right]$$

7. Multinomial Theorem & Its Applications

7.1 Multinomial Theorem :

The expansion of $[x_1 + x_2 + x_3 + ... + x_n]^r$ where n & r are integers $(0 < r \le n)$ is a homogenous expression in $x_1, x_2, x_3, ... x_n$ and given as below :

$[x_1 + x_2 + x_3 + ... + x_n]^r$

$$= \sum\left(\frac{r!}{\lambda_1!\lambda_2!\lambda_3!....\lambda_n!}\right)x_1^{\lambda_2}x_2^{\lambda_2}x_3^{\lambda_3}.....x_n^{\lambda_n}$$

(where n & r are integers $0 \le r \le n$ and

$\lambda_1, \lambda_2, ..., \lambda_n$ are non negative integers)

Such that $\lambda_1 + \lambda_2 + ... + \lambda_n = r$

(valid only if $x_1, x_2, x_3, ... x_n$ are independent of each other)

coefficient of $x_1^{\lambda_1}x_2^{\lambda_2}x_3^{\lambda_3}.....$ = total number of arrangements of r objects out of which λ_1 number of x_1's are identical λ_2 number of x_2's are identical and so on .....

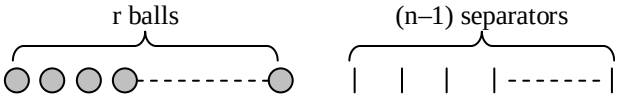
$$= \left(\frac{(\lambda_1+\lambda_2+\lambda_3+.....\lambda_n)!}{\lambda_1!\lambda_2!\lambda_3!.....\lambda_4!}\right) = \frac{r!}{\lambda_1!\,\lambda_2!\,\lambda_3!....\lambda_n!}$$

7.2 Number of distinct terms :

Since $(x_1 + x_2 + x_3 + ... + x_n)^r$ is multiplication of $(x_1 + x_2 + x_3 + ... + x_n)$, r times & will be a homogeneous expansion of r^{th} degree in $x_1, x_2, ...x_n$ So in each term sum of powers of variables must be r

So number of distinct terms will be total number of non-negative integral solution of equation is $\lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n = r$

= Number of ways of distributing r identical objects among n persons



= number of arrangements of r identical balls & n – 1 identical separators.

$$= \frac{(n-1+r)!}{(n-1)!\,r!} = {}^{n+r-1}C_r = {}^{n+r-1}C_{n-1}$$

7.3 Application of multinomial theorem

If we want to distribute n identical objects in r different groups under the condition that empty groups are not allowed.

$a_1 + a_2 + a_3 + ... + a_r = n$

Boundary conditions are $1 \le a_1, a_2 ..... a_r \le n$

(As each box contains at least one object)

Number of ways

= coefficient of x^n in $(x^1 + x^2 + ... + x^n)^r$

= coefficient of x^{n-r} in $(1 + x + x^2 + ... + x^{n-1})^r$

$= {}^{(n-r)+r-1}C_{r-1} = {}^{n-1}C_{r-1}$

8. Divisibility of Numbers

The following chart shows the conditions of divisibility of numbers by 2,3,4,5,6,8,9,25

Divisible by	Condition
2	whose last digit is even (2, 4, 6, 8, 0)
3	sum of whose digits is divisible by 3
4	whose last two digits number is divisible by 4
5	whose last digit is either 0 or 5
6	which is divisible by both 2 and 3
8	whose last three digits number is divisible by 8
9	sum of whose digits is divisible by 9
25	whose last two digits are divisible by 25

9. Sum of Numbers

(i) For given n different digits a_1, a_2, a_3 ....,a_n the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is

$$\mathbf{(a_1 + a_2 + a_3 + ... + a_n)\ (n-1)!}$$

i.e. (sum of the digits) (n-1)!

(ii) Sum of the total numbers which can be formed with given n different digits a_1, a_2, a_3 .....a_n is **$(a_1 + a_2 + a_3 + ... + a_n)(n-1)!.(111 ...n \text{ times})$**

10. Some Important Results About Points

If there are n points in a plane of which m (< n) are collinear, then

(i) Total number of different straight lines obtained by joining these n points is

$$\mathbf{{}^nC_2 - {}^mC_2 + 1}$$

(ii) Total number of different triangles formed by joining these n points is

$$\mathbf{{}^nC_3 - {}^mC_3}$$

(iii) Number of diagonals in polygon of n sides is

$$\mathbf{{}^nC_2 - n} \text{ i.e. } \frac{n(n-3)}{2}$$

(iv) If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is

$$\mathbf{{}^mC_2 \times {}^nC_2} \text{ i.e. } \frac{mn(m-1)(n-1)}{4}$$

SOLVED EXAMPLES

Ex.1 If $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$, then the value of x is-

(A) 123 (B) 125

(C) 121 (D) None of these

Sol. $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!} \Rightarrow \frac{1}{9!}+\frac{1}{10.9!}=\frac{x}{11.10.9!}$

$\Rightarrow \frac{1}{9!}\left[1+\frac{1}{10}\right]=\left(\frac{x}{11.10}\right).\frac{1}{9!}$

$\Rightarrow 1+\frac{1}{10}=\frac{x}{11.10}$

$\Rightarrow \frac{11}{10}=\frac{x}{11.10} \Rightarrow x = 11.11 = 121$

Ans. [C]

Ex.2 The number of different words (meaningful or meaningless) can be formed by taking four different letters from English alphabets is-

(A) $(26)^4$ (B) 358800

(C) $(25)^4$ (D) 15600

Sol. The first letter of four letter word can be chosen by 26 ways, second by 25 ways, third by 24 ways and fourth by 23 ways. So number of four letter words

$= 26 \times 25 \times 24 \times 23 = 358800$ **Ans. [B]**

Ex.3 If $^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$ then the value of **r** is -

(A) 14 (B) 41 (C) 51 (D) 10

Sol. $\frac{^{56}P_{r+6}}{^{54}P_{r+3}}=\frac{30800}{1}$

$\Rightarrow \frac{56!}{(56-r-6)!}=\frac{(30800)\times 54!}{(54-r-3)!}$

$\Rightarrow 56 \times 55 \times (51- r) = 30800$

$\Rightarrow (51- r)=\frac{30800}{56\times 55} = 10$

$\Rightarrow r = 51 - 10 = 41$ **Ans. [B]**

Ex.4 The number of ways in which 2 vacancies can be filled up by 13 candidates is-

(A) 25 (B) 78 (C) 156 (D) 169

Sol. The no. of ways to fill up 2 vacancies by 13 candidates is- $^{13}P_2 = 13 \times 12 = 156$

Ans. [C]

Ex.5 How many different words beginning with A and ending with L can be formed by using the letters of the word' ANILMANGAL'?

(A) 10080 (B) 40320

(C) 20160 (D) None of these

Sol. After fixing the letters A and L in the first and last places, the total number of available places are 8 and the letters are also 8. Out of these 8 letters there are 2 groups of alike letters.

Therefore no. of words = $\frac{8!}{2!\,2!} = 10080$

Ans.[A]

Ex.6 How many numbers can be formed between 20000 and 30000 by using digits 2, 3, 5, 6, 9 when digits may be repeated?

(A) 125 (B) 24

(C) 625 (D) 1250

Sol. First digit between 20000 and 30000 will be 2 which can be chosen by one way. Every number will be of five digits and all the digits can be anything from the given five digit except first digit. So each digit of the remaining four digits can be chosen in 5 ways

$\therefore$ required numbers $= 1 \times 5 \times 5 \times 5 \times 5 = 625$

Ans. [C]

Ex.7 The number of three letters words can be formed from the letters of word 'SACHIN' when I do not come in any word is-

(A) 120 (B) 60

(C) 24 (D) 48

Sol. There are 6 letters in the given word. Then the number of three letters words from the remaining 5 letters after removing I is-

$= {}^5P_3 = 5 \times 4 \times 3 = 60$

Ans. [B]

Ex.8 The number of numbers lying between 100 and 1000 which can be formed with the digits 0, 1, 2, 3, 4, 5, 6 is-

(A) 180 (B) 216

(C) 200 (D) None of these

Sol. Required numbers will have 3 digits so their total number = $^{7}P_3 - {}^{6}P_2 = 180$

Ans. [A]

Ex.9 How many numbers between 1000 and 4000 (including 4000) can be formed with the digits 0, 1, 2, 3, 4 if each digit can be repeated any number of times?

(A) 125 (B) 275

(C) 375 (D) 500

Sol. Required number will have 4 digits and their thousand digit will be 1 or 2 or 3 or 4. The number of such numbers will be 124, 125, 125 and 1 respectively.

$\therefore$ Total numbers = 124 + 125 + 125 + 1 = 375

Ans. [C]

Ex.10 The number of ways in which 7 girls can be stand in a circle so that they do not have the same neighbour in any two arrangements?

(A) 720 (B) 380

(C) 360 (D) None of these

Sol. Seven girls can keep stand in a circle by $\frac{(7-1)!}{2!}$ number of ways, because there is no difference in anticlockwise and clockwise order of their standing in a circle.

$\therefore \frac{(7-1)!}{2!} = 360$ **Ans. [C]**

Ex.11 The number of ways in which 7 men and 7 women can sit on a circular table so that no two women sit together is

(A) 7! . 7! (B) 7! . 6!

(C) $(6!)^2$ (D) 7!

Sol. Here one women will sit between two men. Now fixing the place of one man the remaining 6 men on the circular table can sit in 6! ways. Since there are seven places between 7 men. Therefore seven women can sit on these places in 7! ways.

Thus 7 men and 7 women under the given condition can sit in 7! . 6! ways.

Ans. [B]

Ex.12 $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$ is equal to -

(A) $^{51}C_4$ (B) $^{52}C_4$

(C) $^{53}C_4$ (D) None of these

Sol. The given expression can be written as

$\sum_{r=1}^{5} {}^{52-r}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

[We know that $^{n+1}C_{r+1} = {}^{n}C_r + {}^{n}C_{r+1}$]

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$ **Ans. [B]**

Ex.13 A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. The number of ways in which he can make up his choice is-

(A) 100 (B) 200 (C) 300 (D) 400

Sol. Let there be two groups A and B each containing 5 questions. Questions to be attempted is 6, but not more than 4 from any group. The candidate can select the questions in following ways:

(i) 4 from group A and 2 from group B.

(ii) 3 from group A and 3 from group B.

(iii) 2 from group A and 4 from group B

The number of selections in the above cases are $^{5}C_4 \times {}^{5}C_2$, $^{5}C_3 \times {}^{5}C_3$, $^{5}C_2 \times {}^{5}C_4$ respectively.

$\therefore$ Number of ways of selecting 6 questions

$= {}^{5}C_4 \times {}^{5}C_2 + {}^{5}C_3 \times {}^{5}C_3 + {}^{5}C_2 \times {}^{5}C_4$

= 50 + 100 + 50 = 200 **Ans. [B]**

Ex.14 In how many ways can a committee consisting of one or more members be formed out of 12 members of the Municipal Corporation -

(A) 4095 (B) 5095 (C) 4905 (D) 4090

Sol. Required number of ways

$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \ldots + {}^{12}C_{12} = 2^{12} - 1$

$= 4096 - 1 = 4095$ **Ans. [A]**

Ex.15 Out of 10 white, 9 black and 7 red balls, the number of ways in which selection of one or more balls can be made, is-
(A) 881 (B) 891 (C) 879 (D) 892

Sol. The required number of ways are
$(10 + 1)(9 + 1)(7 + 1) - 1$
$= 11 \times 10 \times 8 - 1 = 879$ **Ans. [C]**

Ex.16 The number of words which can be formed taking 4 different letters out of the letters of the word 'ASSASSINATION', is-
(A) $^{13}C_4 . 4!$ (B) $^{6}C_4 . 4!$
(C) $^{13}P_4 / 2!$ (D) None of these

Sol. Total No. of selections of 4 different letters = $^{6}C_4$
$\therefore$ Total no. of different words = $^{6}C_4 . 4!$
Ans. [B]

Ex.17 There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one each box, could be placed such that a ball does not go to box of its own colour is-
(A) 8 (B) 7
(C) 9 (D) None of these

Sol. Number of derangements are
$= 4! \left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\} = 12 - 4 + 1 = 9$
{Since number of derangements in such a problems is given by n!
$\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - (-1)^n \frac{1}{n!}\right\}$ **Ans. [C]**

Ex.18 The number of 4 digit numbers divisible by 5 which can be formed by using the digits 0, 2, 3, 4, 5 is-
(A) 36 (B) 42
(C) 48 (D) None of these

Sol. For a number to be divisible by 5, unit place should be occupied by 0 or 5-
(i) If unit place is 0 then remaining 3 places can be filled by $^{4}P_3$ ways = 24
(ii) If unit place is 5 then no. of ways
$= 4! - 3! = 18$
$\therefore$ Total number of ways
$= 24 + 18 = 42$ **Ans. [B]**

Ex.19 The sum of all 5 digit numbers which can be formed using digits 1, 2, 3, 4, 5 is-
(A) 6666666 (B) 6600000
(C) 3999960 (D) None of these

Sol. Using formula (Here n = 5)
Sum = $(1+ 2 + 3 + 4 + 5)\, 4!\, (11111)$
$= 15 \times 24 \times 11111 = 3999960$
Ans. [C]

Ex.20 The number of diagonals in an octagon are -
(A) 28 (B) 48
(C) 20 (D) None of these

Sol. Here n = 8 (given)
The number of diagonals are given by
$= \frac{n(n-3)}{2}$
$\Rightarrow \frac{8(8-3)}{2} = \frac{8.5}{2} = 20$ **Ans. [C]**

Ex.21 To fill up 12 vacancies, there are 25 candidates of which 5 are from SC. If 3 of these vacancies are reserved for SC candidates while the remaining are open to all; then the number of ways in which the selection can be made is-
(A) $^{5}C_3 \times {}^{15}C_9$ (B) $^{5}C_3 \times {}^{22}C_9$
(C) $^{5}C_3 \times {}^{20}C_9$ (D) None of these

Sol. 3 vacancies for SC candidates can be filled up from 5 candidates in $^{5}C_3$ ways.
After this for remaining 12 – 3 = 9 vacancies, there will be 25 – 3 = 22 candidates. These vacancies can be filled up in $^{22}C_9$ ways.
Hence required number of ways = $^{5}C_3 \times {}^{22}C_9$
Ans. [B]

Ex.22 Out of 10 given points 6 are in a straight line. The number of the triangles formed by joining any three of them is-
(A) 100 (B) 150
(C) 120 (D) None of these

Sol. A triangle can be formed by joining three points, so there will be $^{10}C_3$ triangles joining any three out of 10 points. But 6 of these 10 points are collinear so these 6 points will give no triangle. Hence the required number of triangles $= {}^{10}C_3 - {}^{6}C_3$
$= 120 - 20 = 100$
Ans. [A]

Ex.23 The number of ways in which 5 biscuits can be distributed among two children is-

(A) 32 (B) 31

(C) 30 (D) None of these

Sol. Each biscuit can be distributed in 2 ways. Therefore number of ways of distributing the biscuits

$= 2^5 = 32$

Now number of ways in which either of the two children does not get any biscuit = 2.

Required number of ways of distribution

$= 32 - 2 = 30$ **Ans. [C]**

Ex.24 How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word 'EQUATION' so that the two consonants occur together?

(A) 1380 (B) 1420

(C) 1440 (D) none

Sol. There are 5 vowels and 3 consonants in the word 'EQUATION'. Three vowels out of 5 and 2 consonants out of 3 can be chosen in ${}^5C_3 \times {}^3C_2$ ways. So, there are ${}^5C_3 \times {}^3C_2$ groups each containing two consonants and three vowels. Now, each group contains 5 letters which are to be arranged in such a way that 2 consonants occur together. Considering 2 consonants as one letter, we have 4 letters which can be arranged in 4! ways. But two consonants can be put together in 2! ways. Therefore, 5 letters in each group can be arranged in 4! × 2! ways. Hence, the required number of words =

$({}^5C_3 \times {}^3C_2) \times 4! \times 2! = 1440$ **Ans. [C]**

Ex.25 Find the total number of proper factors of 7875.

(A) 20 (B) 22

(C) 24 (D) None of these

Sol. We have : $7875 = 3^2 \times 5^3 \times 7^1$

The total number of ways of selecting some or all out of two 3's, three 5's and one 7's is

$(2+1)\,(3+1)\,(1+1) - 1 = 23$

But this includes the given number itself. Therefore, the required number of proper factors is 22. **Ans. [B]**

Ex.26 The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is-

(A) ${}^7P_2 \cdot 2^5$ (B) ${}^7C_2 \cdot 2^5$

(C) ${}^7C_2 \cdot 5^2$ (D) None of these

Sol. Choose any two of the seven digits (in the seven digit number). This may be done in ${}^7C_2$ ways. Put 2 in these two digits. The remaining 5 digits may be arranged using 1 and 3 in 2^5 ways. So, required number of numbers = ${}^7C_2 \times 2^5$. **Ans. [B]**

Ex.27 How many numbers can be formed with the digits 0,1,2, 3,4,5 which are greater than 3000?

(A) 180 (B) 360

(C) 1380 (D) 1500

Sol. The numbers greater than 3000 may contain 4, 5 or 6 digits. Now the 6 digit numbers can be formed by using all 6 given digits which will be ${}^6P_6$. These numbers include the numbers which starts with 0. Such type of numbers are ${}^5P_5$.

Therefore 6 digit numbers greater than 3000

$= {}^6P_6 - {}^5P_5 = 600$

Similarly 5 digit numbers greater than 3000

$= {}^6P_5 - {}^5P_4 = 600$

Now 4 digit numbers which are greater than 3000 should begin with the digit 3, 4 or 5. If the first place of the number is occupied by the digit 3, 4 or 5, then the remaining three places of the number can be filled in ${}^5P_3 = 60$ ways.

Therefore numbers of 4 digits greater than 3000 = 3 x 60 = 180

Hence required numbers

$= 600 + 600 + 180 = 1380$ **Ans. [C]**

Ex.28 In how many ways the letters AAAAA, BBB, CCC, D, EE, F can be arranged in a row when the letter C occur at different places?

(A) $\frac{12!}{5!\,3!\,2!} \times {}^{13}C_3$ (B) $\frac{12!}{5!\,3!\,2!} \times {}^{13}P_3$

(C) $\frac{13!}{5!\,3!\,2!\,3!}$ (D) None of these

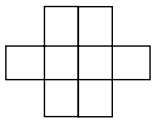
Sol. After removing three C from the given letters, we get 12 letters which can be arranged in a row in $\frac{12!}{5!\,3!\,2!}$ ways.

Now there are 13 places where we can write C. This can be done in $^{13}C_3$ ways.

Hence required no. of ways

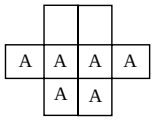
$= \frac{12!}{5!\,3!\,2!} \times {}^{13}C_3$. **Ans. [A]**

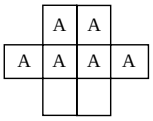
Ex.29 In the given figure of squares, 6 A's should be written in such a manner that every row contains at least one 'A', it is possible in number of ways is-



(A) 24 (B) 25 (C) 26 (D) 27

Sol. There are 8 squares and 6 'A' in given figure. First we can put 6 'A' in these 8 squares by $^8C_6$ number of ways.

(I) 

(II) 

According to question, atleast one 'A' should be included in each row. So after subtracting these two cases, number of ways are

$= (^8C_6 - 2) = 28 - 2 = 26$ **Ans. [C]**

Ex.30 How many words can be formed taking 4 letters from the given 7 capital, 3 vowels and 5 consonants so that each word starts with a capital and contains atleast one vowel ?

(A) 276 (B) 322

(C) 1932 (D) None of these

Sol. Here we have to fill 4 places. The first place can be filled in 7 ways as any one of 7 capital letters can be written at this place.

Now the remaining three places are to be filled in with the help of 3 vowels and 5 consonants so that at least one vowel is always included. The no. of ways = $^8P_3 - {}^5P_3$

Therefore required no. of words = $7(^8P_3 - {}^5P_3)$

$= 7\,(336\text{-}60) = 7 \times 276 = 1932$

Ans. [C]

Ex. 31 If the letters of the word 'RACHIT' are arranged in all possible ways and these words are written out as in a dictionary, then the rank of this word is-

(A) 365 (B) 702

(C) 481 (D) None of these

Sol. The number of words beginning with A (i.e. in which A comes in first place) is $^5P_5 = 5!$.

Similarly number of words beginning with C is 5!, beginning with H is 5! and beginning with I is also 5!.

Now letters of R, four letters A, C, H, I can occur in 4 (5!) = 480 ways. Now word 'RACHIT' happens to be the first word beginning with R. Therefore the rank of this word = 480 + 1 = 481.

Ans. [C]

EXERCISE -1

Q.1 A box contains two different white balls, three different black balls and four different red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw ?
(A) 129 (B) 84
(C) 64 (D) None of these

Q.2 The number of word groups by taking at least 1 letters of each words 'PATH', 'GROW' and 'SKIN' are-
(A) $15^3 - 1$ (B) 15^3
(C) $16^3 - 1$ (D) 16^3

Q.3 Four dice are rolled. The number of possible outcomes in which at least one die shows 2 is-
(A) 1296 (B) 625
(C) 671 (D) None of these

Q.4 The number of words which can be formed from the letters of the word MAXIMUM, if two consonants cannot occur together, is-
(A) 4! (B) $3! \times 4!$
(C) 7! (D) None of these

Q.5 How many six letter words be made out of the letters of 'ASSIST? In how many words the alphabet S alternates with other letters ?
(A) 120, 6 (B) 720, 12
(C) 120, 12 (D) 720, 24

Q.6 In how many ways the alphabets of the word 'MULTIPLE' can be ordered other than itself, when the order of vowels is not changed ?
(A) 3360 (B) 3359 (C) 6720 (D) 20160

Q.7 The total number of words which can be formed using letters of the word 'FAILURE' so that consonants always occupy odd places, is-
(A) 144 (B) 576
(C) 5040 (D) None of these

Q.8 The number of words formed from letters of the word 'EAMCET' so that no two vowels come together, is-
(A) 360 (B) 144 (C) 72 (D) 54

Q.9 How many 5 digit odd numbers can be formed with the help of digits 0, 2, 3, 4 and 6 (Repetition is not allowed) ?
(A) 18 (B) 24 (C) 96 (D) 120

Q.10 How many four digit numbers from the digits 0, 1, 2, 3 will contain 3 at unit place (Repetition is not allowed)
(A) 6 (B) 18
(C) 4 (D) None of these

Q.11 The number of numbers of 4 digits which are not divisible by 5 are (when repetition is allowed)
(A) 7200 (B) 3600 (C) 14400 (D) 1800

Q.12 How many 5 digit numbers be formed by the digits 1, 2, 3, 4, 5 which are divisible by 4 (Repetition not allowed) ?
(A) 24 (B) 120
(C) 72 (D) None of these

Q.13 The total number of 5- digit numbers formed with the digits 0, 1, 2, 3, 4 and 5 which are divisible by 3, is (Repetition not allowed) -
(A) 216 (B) 240 (C) 600 (D) 3125

Q.14 How many 6 digit different number can formed with help of the digits of numbers 121 and 202?
(A) 25 (B) 50
(C) 100 (D) None of these

Q.15 The number of 4 - digit numbers formed with the digits 1, 2, 3, 4, 5, 6, 7 which are divisible by 25 is (Repetition not allowed) -
(A) 20 (B) 30
(C) 40 (D) None of these

Q.16 The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that terminal digits are even is (repetition is not allowed) -
(A) 72 (B) 720 (C) 144 (D) 288

Q.17 Using all digits 2, 3, 4, 5, 6, how many even numbers can be formed?
(A) 24 (B) 48 (C) 72 (D) 120

Q.18 How many any digits non zero numbers can be formed with the help of digits 0, 1, 2, 3, 4, when no digit is repeated in any number?
(A) 260 (B) 336
(C) 410 (D) None of these

Q.19 How many four digits numbers can be formed with digits 1, 2, 3, 4, 5, 6 when it includes 1 and 2 necessarily (repetition is not allowed) ?
(A) 6 (B) 288
(C) 144 (D) 48

Q.20 The total number of seven digit numbers the sum of whose digits is even is-
(A) 9000000 (B) 4500000
(C) 8100000 (D) None of these

Q.21 The number of times the digit 3 will be written when listing 1 to 1000 is ……
(A) 300 (B) 271
(C) 302 (D) 269

Q.22 The number of ways in which any four letters can be selected from the word 'CORGOO' is-
(A) 15 (B) 11
(C) 7 (D) None of these

Q.23 The total number of ways of selecting five letters from the letters of the word 'INDEPENDENT' is-
(A) 72 (B) 3320
(C) 120 (D) None of these

Q.24 In how many ways can a selection of 4 letters be made out of the letters of the word 'MATHEMATICS'?
(A) 136 (B) 330
(C) 70 (D) None of these

Q.25 The number of words which can be formed by taking two same and two different letters from the letters of the word 'COMBINATION' is-
(A) 756 (B) 1512
(C) 252 (D) None of these

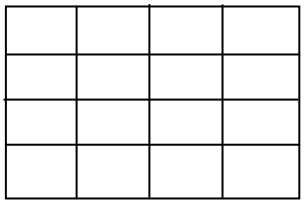
Q.26 Taking three same and one different letters from the letters of the word 'PROPORTION', the number of words which can be formed is-
(A) 18 (B) 360
(C) 20 (D) None of these

Q.27 How many words can be formed by taking three letters from the letters of the word 'SERIES'?
(A) 24 (B) 18
(C) 42 (D) None of these

Q.28 The number of words which can be formed using 4 letters of the word 'EXAMINATION' is-
(A) 1896 (B) 2136
(C) 2454 (D) None of these

EXERCISE -2

Q.1 The number of rectangles in the adjoining figure is -



(A) 5×5
(B) ${}^5P_2 \times {}^5P_2$
(C) ${}^5C_2 \times {}^5C_2$
(D) None of these

Q.2 In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B, and no two are parallel. Then the number of intersection points the lines have is equal to -
(A) 535 (B) 601
(C) 728 (D) None of these.

Q.3 The number of numbers between 1 and 10^{10} which contain the digit 1 is-
(A) $10^{10} - 9^{10} - 1$ (B) 9^{10}
(C) $10^{10} - 8^{10}$ (D) None of these.

Q.4 A set contains $(2n + 1)$ elements. If the number of subsets of this set which contain at most n elements is 4096, then the value of n is-
(A) 6 (B) 15
(C) 21 (D) None of these.

Q.5 How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?
(A) 16 (B) 36
(C) 60 (D) 180

Q.6 All possible two factors products are formed from numbers 1, 2, 3, 4, ..., 200. The number of factors out of the total obtained which are multiples of 5 is-
(A) 5040 (B) 7180
(C) 8150 (D) None of these

Q.7 If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is-
(A) ${}^{n-2}C_3$ (B) ${}^{n-3}C_2$
(C) ${}^{n-3}C_3$ (D) None of these

Q.8 Between two junction stations A and B there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive is-
(A) ${}^8C_4$ (B) ${}^9C_4$
(C) ${}^{12}C_4 - 4$ (D) None of these

Q.9 The number of integral solutions of $x + y + z = 0$ with $x \geq -5$, $y \geq -5$, $z \geq -5$ is-
(A) 135 (B) 136
(C) 455 (D) 105

Q.10 The number of non-negative integral solutions of $x + y + z \leq n$, where $n \in N$ is -
(A) ${}^{n+3}C_3$ (B) ${}^{n+4}C_4$
(C) ${}^{n+5}C_5$ (D) None of these

Q.11 The number of ways is which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question is -
(A) ${}^{21}C_7$ (B) ${}^{21}C_8$
(C) ${}^{21}C_9$ (D) ${}^{21}C_{10}$

Q.12 Number of ways of placing 5 identical balls in 3 identical boxes (no box remains empty), is-
(A) 6 (B) 2
(C) 3 (D) None of these.

Q.13 Number of ways of placing 5 identical balls in 3 different boxes (no box remain empty), is –
(A) 6 (B) 12
(C) 150 (D) None of these.

Q.14 Number of ways of placing 5 different balls in 3 identical boxes (no box remains empty), is-
(A) 50 (B) 10
(C) 25 (D) none of these

Q.15 Number of ways of placing 5 different balls in 3 different boxes (no box remains empty), is-
(A) 10 (B) 15
(C) 25 (D) 150

Q.16 A class has n students. We have to form a team of students including at least 2 students and excluding at least 2 students. The number of ways of forming the team is-
(A) $2^n - 2n$ (B) $2^n - 2n - 2$
(C) $2^n - 2n - 4$ (D) None of these

Q.17 There were two women participating in a chess tournament. Every participant played two games with the other participants. The number of games that the men played among themselves proved to exceed by 66 numbers of games that the men played with the women. The number of participants is-
(A) 6 (B) 11
(C) 13 (D) None

Q.18 The maximum number of points of intersection of five lines and four circles is-
(A) 60 (B) 72
(C) 62 (D) None of these

Q.19 The highest power of 18 contained in $^{50}C_{25}$ is :
(A) 3 (B) 0 (C) 1 (D) 2

Q.20 ABCD is a convex quadrilateral and 3, 4, 5 and 6 points are marked on the sides AB, BC, CD and DA, respectively. The number of triangles with vertices on different sides is-
(A) 270 (B) 220 (C) 282 (D) 342

Q.21 The number of arrangements of all the letters occurring in the phrase 'SUCCESS IN IIT' is
(A) 66528 (B) 665280
(C) 6652800 (D) None of these

Q.22 The number of ways in which n distinct objects can be put into two identical boxes so that no box remains empty, is-
(A) $2^n - 2$ (B) $2^n - 1$
(C) $2^{n-1} - 1$ (D) $n^2 - 2$

Q.23 If a denoted the number of permutations of x + 2 things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of x – 11 things taken all at a time such that $a = 182bc$, then the value of x is-
(A) 15 (B) 12
(C) 10 (D) 18

Q.24 The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is-
(A) $^7P_2 2^5$ (B) $^7C_2\ 2^5$
(C) $^7C_2 5^2$ (D) None of these

Q.25 The letters of word 'ZENITH' are written in all possible ways. If all these words are written in the order of a dictionary, then the rank of the word 'ZENITH' is-
(A) 716 (B) 692 (C) 698 (D) 616

EXERCISE -3

Old Examination Questions [AIEEE/JEE Main]

Q.1 Find the no. of numbers which can be formed with digits 0,1, 2, 3, 4 greater than 1000 and less than 4000 if repetition is allowed- **[AIEEE 2002]**

(A) 125 (B) 400

(C) 375 (D) 374

Q.2 If repetition of the digits is allowed, then the number of even natural numbers having three digits is- **[AIEEE-2002]**

(A) 250 (B) 350

(C) 450 (D) 550

Q.3 If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals- **[AIEEE 2003]**

(A) ${}^{n+1}C_{r+1}$ (B) ${}^{n+2}C_r$

(C) ${}^{n+2}C_{r+1}$ (D) ${}^{n+1}C_r$

Q.4 A student is to answer 10 out of 13 questions, an examination such that he must choose least 4 from the first five questions. The number of choices available to him, is- **[AIEEE 2003]**

(A) 346 (B) 140

(C) 196 (D) 280

Q.5 The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by – **[AIEEE 2003]**

(A) (7!) × (5!) (B) (6!) × (5!)

(C) 30 (D) (5!) × (4!)

Q.6 How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order? **[AIEEE 2004]**

(A) 120 (B) 240 (C) 360 (D) 480

Q.7 The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is- **[AIEEE 2004]**

(A) 5 (B) 21 (C) 3^8 (D) ${}^8C_3$

Q.8 If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number - **[AIEEE-2005]**

(A) 601 (B) 600 (C) 603 (D) 602

Q.9 The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is - **[AIEEE-2005]**

(A) ${}^{55}C_4$ (B) ${}^{55}C_3$ (C) ${}^{56}C_3$ (D) ${}^{56}C_4$

Q.10 At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is - **[AIEEE 2006]**

(A) 6210 (B) 385 (C) 1110 (D) 5040

Q.11 The set S : {1, 2, 3, …, 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is- **[AIEEE 2007]**

(A) $12!/3!(4!)^3$ (B) $12!/3!(3!)^4$

(C) $12!/(4!)^3$ (D) $12!/(3!)^4$

Q.12 How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which not two S are adjacent ? **[AIEEE 2008]**

(A) 6. 7. ${}^8C_4$ (B) 6. 8. ${}^7C_4$

(C) 7. ${}^6C_4$. ${}^8C_4$ (D) 8. ${}^6C_4$. ${}^7C_4$

Q.13 In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1:

The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$

Statement -2:

The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. **[AIEEE 2008]**

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is true, Statement -2 is false
(D) Statement-1 is false, Statement-2 is true

Q.14 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is- **[AIEEE 2009]**
(A) Less than 500
(B) At least 500 but less than 750
(C) At least 750 but less than 1000
(D) At least 1000

Q.15 There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is – **[AIEEE 2010]**
(A) 3 (B) 36 (C) 66 (D) 108

Q.16 **Statement-1 :**
The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^{9}C_{3}$.
Statement-2:
The number of ways of choosing any 3 places from 9 different places is $^{9}C_{3}$.
[AIEEE 2011]
(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false.
(D) Statement-1 is false, Statement-2 is true.

Q.17 If seven women and seven men are to be seated around a circular table such that there is a man on either side of every woman, then the number of seating arrangements is : **[AIEEE Online - 2012]**
(A) 7! (B) 6! 7! (C) $(6!)^2$ (D) $(7!)^2$

Q.18 The number of arrangements that can be from the letters a, b, c, d, e and f , taken 3 at a time without repetition and each arrangement containing at least one vowel, is **[AIEEE Online - 2012]**
(A) 72 (B) 96 (C) 24 (D) 128

Q.19 Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$ then the value of n is – **[JEE Main - 2013]**
(A) 10 (B) 8 (C) 7 (D) 5

Q.20 Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of A × B having 3 or more elements is – **[JEE Main - 2013]**
(A) 219 (B) 211 (C) 256 (D) 220

Q.21 A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is : **[JEE Main Online - 2013]**
(A) 40 (B) 41 (C) 16 (D) 32

Q.22 The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is – **[JEE Main Online - 2013]**
(A) $^{30}C_7$ (B) $^{21}C_8$ (C) $^{21}C_7$ (D) $^{30}C_8$

Q.23 On the sides AB, BC, CA of a ΔABC, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are : **[JEE Main Online - 2013]**
(A) 210 (B) 205 (C) 215 (D) 220

Q.24 5-digit numbers are to be formed using 2, 3, 5, 7, 9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then p : q is :

[JEE Main Online - 2013]

(A) 6 : 5 (B) 3 : 2 (C) 4 : 3 (D) 5 : 3

Q.25 The sum of the digits in the unit's place of all the 4-digit numbers formed by using the numbers 3, 4, 5 and 6, without repetition is :

[JEE Main Online - 2014]

(A) 432 (B) 108 (C) 36 (D) 18

Q.26 Two women and some men participated in a class tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval:

[JEE Main Online - 2014]

(A) [8, 9] (B) [10, 12)

(C) (11, 13] (D) (14, 17)

Q.27 8 – digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do not occupy odd places, is : **[JEE Main Online - 2014]**

(A) 160 (B) 120 (C) 60 (D) 48

Q.28 An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is : **[JEE Main Online - 2014]**

(A) 72 (7!) (B) 18 (7!)

(C) 40 (7!) (D) 36 (7!)

Q.29 The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[JEE Main - 2015]**

(A) 216 (B) 192 (C) 120 (D) 72

Q.30 If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is : **[JEE Main Online- 2015]**

(A) 10 (B) 12 (C) 9 (D) 6

Q.31 If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[JEE Main - 2016]**

(A) 46^{th} (B) 59^{th} (C) 52^{nd} (D) 58^{th}

Q.32 If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is -

[JEE Main Online - 2016]

(A) $\frac{11!}{(2!)^3}$ (B) 110 (C) 56 (D) 59

Q.33 An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is :

[JEE Main Online - 2016]

(A) $\frac{240}{729}$ (B) $\frac{192}{729}$ (C) $\frac{256}{729}$ (D) $\frac{496}{729}$

Q.34 A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

[JEE Main - 2017]

(A) 468 (B) 469

(C) 484 (D) 485

Q.35 If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}, then the probability that their sum as well as absolute difference are both multiple of 4, is : **[JEE Main - 2017]**

(A) $\frac{12}{55}$ (B) $\frac{14}{45}$ (C) $\frac{7}{55}$ (D) $\frac{6}{55}$

Q.36 If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is : **[JEE Main Online - 2017]**

(A) 47^{th} (B) 45^{th}

(C) 46^{th} (D) 44^{th}

Q.37 The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1, and a particular girl G_1 never sit adjacent to each other, is - **[JEE Main Online - 2017]**

(A) 7! (B) $5 \times 6!$

(C) $6 \times 6!$ (D) $5 \times 7!$

Q.38 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is : **[JEE Main - 2018]**

(A) at least 1000

(B) less than 500

(C) at least 500 but less than 750

(D) at least 750 but less than 1000

Q.39 n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is **[JEE-Main Online-2018]**

(A) 6 (B) 8 (C) 9 (D) 7

Q.40 The number of four letter words that can be formed using the letters of the word BARRACK is- **[JEE-Main Online-2018]**

(A) 144 (B) 120 (C) 264 (D) 270

Q.41 The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4, (repetition of digits is not allowed) and are multiple of 3 is ? **[JEE-Main Online-2018]**

(A) 30 (B) 48

(C) 24 (D) 36

EXERCISE-4

Old Examination Questions [IIT JEE Advanced]

Q.1 A polygon has 44 diagonals, then the number of its sides are - **[IIT-1993]**
(A) 11 (B) 7
(C) 8 (D) none of these

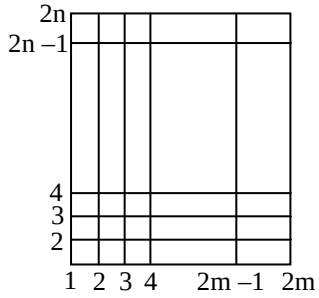
Q.2 An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5, and 7. The smallest value of n for which this is possible is - **[IIT 1998]**
(A) 6 (B) 7 (C) 8 (D) 9

Q.3 ${}^{n}C_{r} + 2{}^{n}C_{r+1} + {}^{n}C_{r+2}$ is equal to $(2 \le r \le n)$ **[IIT(s) 2000]**
(A) $2 \cdot {}^{n}C_{r+2}$ (B) ${}^{n+1}C_{r+1}$
(C) ${}^{n+2}C_{r+2}$ (D) none of these

Q.4 The number of arrangement of the letters of the word BANANA in which the two N's do not appear adjacently is - **[IIT Scr- 2002]**
(A) 40 (B) 60 (C) 80 (D) 100

Q.5 No. of points with integer coordinates lie inside the triangle whose vertices are (0, 0), (0, 21), (21, 0) is: **[IIT Scr. 2003]**
(A) 190 (B) 185 (C) 210 (D) 230

Q.6 A rectangle has sides of $(2m - 1)$ & $(2n - 1)$ units as shown in the figure composed of squares having edge length one unit then no. of rectangles which have odd unit length **[IIT Scr. 2005]**



(A) $m^2 - n^2$ (B) $m(n + 1)\, n(n + 1)$
(C) 4^{m+n-2} (D) m^2n^2

Q.7 If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is – **[IIT-2006]**
(A) 224 (B) 225
(C) 252 (D) 256

Q.8 The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is **[IIT- 2007]**
(A) 360 (B) 192
(C) 96 (D) 48

Q.9 The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digit 1, 2 and 3 only, is **[IIT- 2009]**
(A) 55 (B) 66
(C) 77 (D) 88

Q.10 Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to **[IIT-2010]**
(A) 25 (B) 34
(C) 42 (D) 41

Q.11 The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is **[IIT-2012]**
(A) 75 (B) 150
(C) 210 (D) 243

Q.12 A debate club consist of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is - **[JEE Advance - 2016]**
(A) 380 (B) 320
(C) 260 (D) 95

Q.13 Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated, and let y be the number of such words where exactly one letter is repeated twice and no other is repeated. Then, $\frac{y}{9x}$ = **[JEE-Advanced 2017]**

Q.14 Let S = {1, 2, 3, ... 9}. For k = 1, 2, ... 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5$ = **[JEE-Advanced 2017]**

(A) 126 (B) 125 (C) 210 (D) 252

Q.15 The number of 5 digit number which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _____. **[JEE - Advance 2018]**

Q.16 In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total numbers of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

[JEE - Advance 2018]

List-I	**List-II**
(P) The value of α_1 is	(1) 136
(Q) The value of α_2 is	(2) 189
(R) The value of α_3 is	(3) 192
(S) The value of α_4 is	(4) 200
	(5) 381
	(6) 461

The correct option is :

(A) P → 4; Q → 6; R → 2; S → 1

(B) P → 1; Q → 4; R → 2; S → 3

(C) P → 4; Q → 6; R → 5; S → 2

(D) P → 4; Q → 2; R → 3; S → 1

ANSWER KEY

EXERCISE -1

1. (C)	**2.** (B)	**3.** (C)	**4.** (A)	**5.** (C)	**6.** (B)	**7.** (B)
8. (C)	**9.** (A)	**10.** (C)	**11.** (A)	**12.** (A)	**13.** (A)	**14.** (B)
15. (C)	**16.** (B)	**17.** (C)	**18.** (A)	**19.** (C)	**20.** (B)	**21.** (A)
22. (C)	**23.** (A)	**24.** (A)	**25.** (A)	**26.** (C)	**27.** (C)	**28.** (C)

EXERCISE -2

1. (C)	**2.** (A)	**3.** (A)	**4.** (A)	**5.** (C)	**6.** (B)	**7.** (A)
8. (B)	**9.** (B)	**10.** (A)	**11.** (A)	**12.** (B)	**13.** (A)	**14.** (C)
15. (D)	**16.** (B)	**17.** (C)	**18.** (C)	**19.** (C)	**20.** (D)	**21.** (C)
22. (C)	**23.** (B)	**24.** (B)	**25.** (D)			

EXERCISE -3

1. (D)	**2.** (C)	**3.** (C)	**4.** (C)	**5.** (B)	**6.** (C)	**7.** (B)
8. (A)	**9.** (D)	**10.** (B)	**11.** (C)	**12.** (C)	**13.** (D)	**14.** (D)
15. (D)	**16.** (A)	**17.** (B)	**18.** (B)	**19.** (D)	**20.** (A)	**21.** (B)
22. (C)	**23.** (B)	**24.** (D)	**25.** (B)	**26.** (B)	**27.** (B)	**28.** (D)
29. (B)	**30.** (B)	**31.** (D)	**32.** (D)	**33.** (C)	**34.** (D)	**35.** (D)
36. (C)	**37.** (B)	**38.** (A)	**39.** (D)	**40.** (D)	**41.** (A)	

EXERCISE -4

1. (A)	**2.** (B)	**3.** (C)	**4.** (A)	**5.** (A)	**6.** (D)	**7.** (B)
8. (C)	**9.** (C)	**10.** (D)	**11.** (B)	**12.** (A)	**13.** 5	**14.** (A)
15. 625.00	**16.** (C)					

All exercise's detailed solutions are available at www.careerpoint.ac.in

SOLUTIONS

EXERCISE-4

1.[A] In a polygon of 'n' sides
No. of diagonals = ${}^nC_2 - n = 44$

$\Rightarrow \frac{n(n-1)}{2} - n = 44$

$\Rightarrow n(n-3) = 88$

$\Rightarrow n(n-3) = 11.8$

$\therefore n = 11$

2.[B] No. of the form X X X X
No. of numbers = 3. 3. 3. 3
$= 3^n \geq 900$

$\because 3^6 = 729$

$3^7 = 2187 \qquad \therefore n_{min} = 7$

3.[C] ${}^nC_r + 2.\ {}^nC_{r+1} + {}^nC_{r+2}$

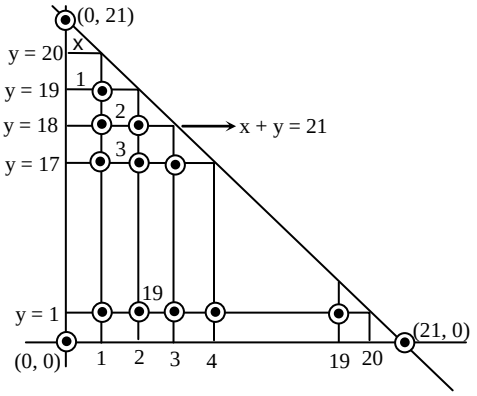
$= \underbrace{{}^nC_r + {}^nC_{r+1}}_{{}^{n+1}C_{r+1}} + \underbrace{{}^nC_{r+1} + {}^nC_{r+2}}_{{}^{n+1}C_{r+2}}$

$= {}^{n+2}C_{r+2}$

4.[A] We have B, (AAA), (NN)
Excluding (NN), rest can be arranged in
$\frac{4!}{3!} = 4$ ways

Now • B • A • A • A •
(NN) can be arranged at dotted places in
${}^5C_2 \cdot 1 = 10$ ways
$\therefore$ no. of words = 4.10 = 40

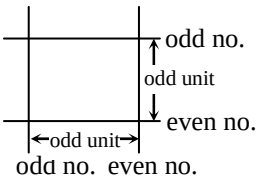
5.[A]



$\therefore$ No. of points = 1 + 2 + 3 ..... + 19

$= \frac{19.20}{2} = 190$

6.[D]



No. of rectangle = $({}^mC_1 \times {}^mC_1) \times ({}^nC_1 \times {}^nC_1)$
$= m^2n^2$

7.[B] If LCM (p, q) = $r^a . s^b . t^c$
then no. of order pairs
= (2a + 1). (2b + 1) . (2c + 1)
= 5. 9. 5
= 225

8.[C] In order C, C, H, I, N, O
In dictionary:
C C __ __ __ __ $\rightarrow$ 4!
C H __ __ __ __ $\rightarrow$ 4!
C I __ __ __ __ $\rightarrow$ 4!
C N __ __ __ __ $\rightarrow$ 4!
C O C H I N
$\therefore$ No. of words before COCHIN = 4 . 4!
= 96

9.[C] = Coefficient of x^{10} in $(x^1 + x^2 + x^3)^7$
= Coefficient of x^{10} in $x^7(1 + x + x^2)^7$
= Coefficient of x^3 in $(1 + x + x^2)^7$
$= {}^7C_0 + {}^7C_1(x + x^2) + {}^7C_2(x + x^2)^2$
$+ {}^7C_3(x + x^2)^3 ..... + {}^7C_7(x + x^2)^7$
Coefficient of $x^3 = 0 + 0 + {}^7C_2 . 2 + {}^7C_3 + 0... + 0$

$= 21 . 2 + \frac{7.6.5}{1.2.3}$

$= 42 + 35 = 77$

10.[D] S = {1, 2, 3, 4}

Possible subset no. of elements in ways		
Set A	**Set B**	
0	0	= 1
1	0	$= {}^4C_1 = 4$
2	0	$= {}^4C_2 = 6$

1	1	$= {}^4C_2 = 6$
3	0	$= {}^4C_3 = 4$
2	1	$= {}^4C_2\,{}^2C_1 = 12$
4	0	$= {}^4C_4 = 1$
3	1	$= \frac{4!}{3!\,1!} = 4$
2	2	$= \frac{4!}{2!\,2!\,2!} = 3$

Total = 1 + 4 + 6 + 6 + 4 + 12 + 1 + 4 + 3 = 41

11. [B]

G_1	G_2	G_3
1	**1**	**3**
1	**2**	**2**

$$\left(\frac{5!}{1!\,1!\,3!\,2!}+\frac{5!}{1!\,2!\,2!\,2!}\right)3!$$

= 150

12.[A] Total No. of ways

$= ({}^4C_0 \times {}^6C_4 + {}^4C_1 \times {}^6C_3) \times {}^4C_1$

$= (15 + 80) \times 4$

= 380

13.[5] $x = \lfloor 10$

$y = {}^{10}C_1 \times {}^9C_8 \times \frac{\lfloor 10}{\lfloor 2}$

$= 45 \times \lfloor 10$

$\therefore \frac{y}{9x} = \frac{45 \times \lfloor 10}{9 \times \lfloor 10} = 5$

14.[A] $N_1 = {}^5C_1 \times {}^4C_4 = 5$

$N_2 = {}^5C_2 \times {}^4C_3 = 40$

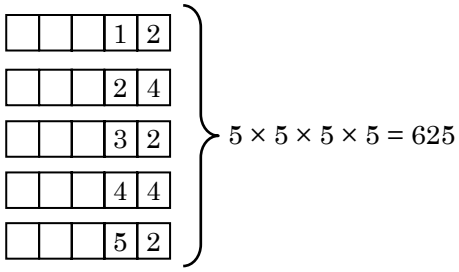
$N_3 = {}^5C_3 \times {}^4C_2 = 60$

$N_4 = {}^5C_4 \times {}^4C_1 = 20$

$N_5 = {}^5C_5 = 1$

$\therefore N_1 + N_2 + N_3 + N_4 + N_5 = 126$

15. [625.00]



16.[C] (i) $\alpha_1 = {}^6C_3 \times {}^5C_2 = 200$

(ii) $\alpha_2 = {}^6C_1 \times {}^5C_1 + {}^6C_2 \times {}^5C_2 + {}^6C_3 \times {}^5C_3 + {}^6C_4 \times {}^5C_4 + {}^6C_5 \times {}^5C_5 = {}^{11}C_5 - 1 = 461$

(iii) $\alpha_3 = {}^5C_2 \times {}^6C_3 + {}^5C_3 \times {}^6C_2 + {}^5C_4 \times {}^6C_1 + {}^5C_5 \times {}^6C_0 = 200 + 150 + 30 + 1 = 381$

(iv) $\alpha_4 = {}^5C_2 \times {}^6C_2 - {}^4C_1 \times {}^5C_1 + {}^5C_3 \times {}^6C_1 - {}^4C_2 \times {}^1C_1 + {}^5C_1 + {}^5C_4 = 150 - 20 + 60 - 6 + 5 = 189$