

STUDY MATERIAL **JEE**

FOR MAIN & ADVANCED

PHYSICS

CHEMISTRY

MATHEMATICS





CAREER POINT

Study Material for JEE Main & Advanced preparation

Prepared by Career Point Kota Experts

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PHYSICS

Class 11

Mechanics (Part-I)

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Class 12

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CHEMISTRY

Class 11

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Class 12

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MATHEMATICS

Class 11

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Class 12

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- ♦ Maxima & Minima

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- ♦ Complex Number
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- ♦ Determinants
- ♦ Matrices
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Note to the Students

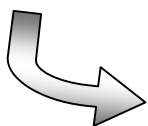
Career Point offers this must have Study Package in Physics, Chemistry and Mathematics to meet the complete curriculum needs of engineering aspirants. The set comprises of 18 books: **Physics** - set of 3 books for class 11 and set of 3 books for Class 12; **Chemistry** - set of 3 books for class 11 and set of 3 books for Class 12 and **Mathematics** - set of 3 books for class 11 and set of 3 books for Class 12. The set caters to the different requirements of students in classes XI and XII. It offers complete and systematic coverage of **JEE Main** and **JEE Advanced** syllabi and aims to provide firm foundation in learning and develop competitive edge in preparation of the JEE and other engineering entrance examinations.

COMPONENTS OF EACH CHAPTER

These books are designed with an engaging and preparation-focused pedagogy and offer a perfect balance of conceptual learning and problem solving skills.

Theory & Concepts

Each chapter consists of high quality theory that covers all the topics, sub-topics and concepts of JEE syllabus.

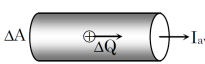


Current Electricity

KEY CONCEPT

- Electric Current**

It is the time rate of flow of charge through a conductor when there is net transfer of charge (say ΔQ) across a cross section during a time interval (say Δt), we define average electric current as

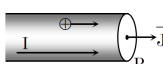

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

1.1 The instantaneous current at any instant 't' as

$$I_{in} = \frac{dq}{dt}$$
- Current Density**

2.1 The current density at a point is defined as a vector having magnitude equal to current per unit area surrounding that point and normal to the direction of charge flow. It is represented by \vec{J} .

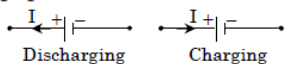
2.2 At a point P if current I passes normally through area dA (see figure), then

$$\vec{J} = \frac{d\vec{I}}{dA}$$


Important Points

This part contains important concepts & formulas of chapter at one place in short manner, So that student can revise all these in short time.

Important Points

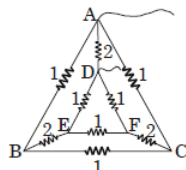
- Electric current, though it has a sense of direction, is a scalar quantity but current density is a vector.
- Electrical resistance of conductor increases with temperature while that of semiconductor or insulator decreases.
- When current in a cell flows from -ve to +ve terminal it is said to be discharging otherwise charging.
- The resistance of ideal ammeter should be zero (or very less in case of good ammeters) and the resistance of ideal voltmeter should be infinite (or very high in case of good voltmeters)
- The best device for measuring potential difference is potentiometer as it measures open loop potential difference.
- During charging of RC circuit the charge grows and during discharging charge decays but current always decays in the branch containing capacitor of circuit. Steady state current in a branch having capacitor is always zero with

Solved Examples (JEE Main/Advanced)

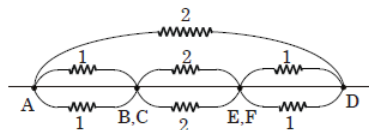
To understand the application of concepts, there is a solved example section. It contains large variety of all types of solved examples with explanation to ensure understanding the application of concepts.

SOLVED EXAMPLES

Ex.1 A network of nine conductors connects six points A, B, C, D, E and F as shown. The figures denote resistances in ohms. The equivalent resistance between A and D is



Sol. B and C are equipotential points and so are E and F. Here the circuit can be redrawn as shown in Figure. 1Ω and 1Ω in parallel sum up to $1/2\Omega$; 2Ω and 2Ω in parallel sum up to 1Ω ; $1/2\Omega$, 1Ω , $1/2\Omega$ in series sum up to $1/2 + 1 + 1/2 = 2\Omega$; 2Ω and 2Ω in parallel sum up to $= 1\Omega$.



$$\text{and } 3y + 1y = 6$$

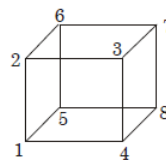
$$\text{or } y = 1.5 \text{ A}$$

$$V_{AB} = \sum r - \sum e = (-2 \times 0.5 + 3 \times 1.5) - 4 = -0.5 \text{ V}$$

The minus sign shows that B is at a higher potential than A. Thus

$$V_{BA} = -1 \text{ V}$$

Ex.3 Find the resistance of a wire frame shaped as a cube when measured between points 1 and 7. The resistance of each edge is r .



Sol. Symmetry about entrance point 1 and exit point 7 shows that 2, 4, 5 are equipotential points and 5, 6, and 8 are equipotential points. Hence the circuit can be redrawn as shown in Figure. The resistance r , r and r in parallel sum up to $r/3$.

Practice Exercises

Exercise Level - 1 : It contains objective questions with single correct choice to ensure sufficient practice to accurately apply formulae and concepts.

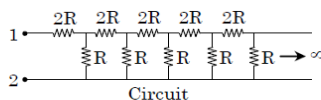
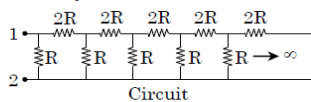
Exercise Level - 2 : It contains single objective type questions with moderate difficulty level to enhance the conceptual and application level of the student.

Exercise Level - 3 : It contains all variety of questions as per level of JEE Advanced such as MCQ, Column match, Passage based & Numerical type etc.

EXERCISE (Level-3)

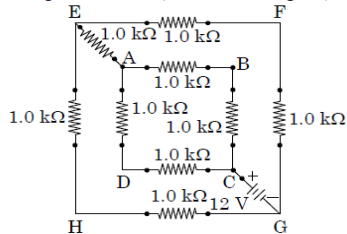
Part-A: Multiple correct answer type questions

Q.1 Two circuits (as shown in figure) are called circuit A and circuit B. The equivalent resistance of circuit A is x and that of circuit B is y between 1 and 2.



- (A) $y > x$ (B) $y = (\sqrt{3} + 1) R$
(C) $xy = 2R^2$ (D) $y - x = 2R$

Q.3 In the given circuit (as shown in figure)



- (A) the equivalent resistance between C and G is $3 \text{ k}\Omega$.
(B) the current provided by the source is 4 mA .
(C) the current provided by the source is 8 mA .
(D) voltage across points G and E is 4 V .

Exercise Level - 4 : It contains previous years JEE Main/Advanced questions from Year 2005 to 2018.

EXERCISE (Level-4)
 Old Examination Questions

Section-A [JEE Main]

Q.1 A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be - [AIEEE-2005]
 (A) 10^3 (B) 10^5 (C) 99995 (D) 9995

Q.2 In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be- [AIEEE-2005]

(A) 200 Ω (B) 100 Ω (C) 500 Ω (D) 1000 Ω

Q.5 In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is - [AIEEE-2005]
 (A) 1 Ω (B) 0.5 Ω (C) 4 Ω (D) 2 Ω

Q.6 A material 'B' has twice the specific resistance of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. Then for the two wires to have the same resistance, the ratio ℓ_B/ℓ_A of their respective lengths must be - [AIEEE-2006]
 (A) $\frac{1}{4}$ (B) 2 (C) 1 (D) $\frac{1}{2}$

Q.7 The Kirchhoff's first law ($\sum i = 0$) and second law ($\sum iR = \sum E$), where the symbols have usual meanings, are respectively based on - [AIEEE-2006]

Exercise Level - 5 : Advanced level a bit complex quesiotns for students for solid rock preperetion for Top Rankers.

Answer key

Ankswer key is provided at the end of the exercise sheets.

ANSWER KEY						
EXERCISE (Level-1)						
1. (B)	2. (C)	3. (A)	4. (B)	5. (B)	6. (C)	7. (C)
8. (C)	9. (B)	10. (D)	11. (B)	12. (B)	13. (C)	14. (A)
15. (A)	16. (C)	17. (C)	18. (D)	19. (B)	20. (A)	21. (A)
22. (B)	23. (C)	24. (A)	25. (B)	26. (B)	27. (B)	28. (B)
29. (B)	30. (B)	31. (C)	32. (D)	33. (B)	34. (A)	35. (C)
36. (B)	37. (B)	38. (C)	39. (B)	40. (D)	41. (C)	42. (C)

Revision Plan

We emphasis that every student should prepare his/her own revision plan. For this purpose there is Revision Plan Section in each chapter which student should prepare while going thorough the study material. This will be useful at the time of final revision before final exam for quick & effective revision.

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

- A. Write Question Number (QN) which you are unable to solve at your own in **column A**.
- B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.
- C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		

Online Solutions

Self explanatory and detailed solution of all exercises above are available on Career Point website www.careerpoint.ac.in

CURRENT ELECTRICITY

EXERCISE (Level-1)

Answer Key & Solution

Question Number	Solution	Question Number	Solution	Question Number	Solution	Question Number	Solution
1	Click Here	12	Click Here	23	Click Here	34	Click Here
2	Click Here	13	Click Here	24	Click Here	35	Click Here
3	Click Here	14	Click Here	25	Click Here	36	Click Here
4	Click Here	15	Click Here	26	Click Here	37	Click Here
5	Click Here	16	Click Here	27	Click Here	38	Click Here
6	Click Here	17	Click Here	28	Click Here	39	Click Here
7	Click Here	18	Click Here	29	Click Here	40	Click Here
8	Click Here	19	Click Here	30	Click Here	41	Click Here
9	Click Here	20	Click Here	31	Click Here	42	Click Here
10	Click Here	21	Click Here	32	Click Here		
11	Click Here	22	Click Here	33	Click Here		

Sol.1 [B]

Given

$$I = 1.1 \text{ A}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$A = \pi r^2 = \pi \times (0.05)^2$$

$$= 78.5 \times 10^{-4} \text{ cm}^2$$

$$v_d = \frac{I}{neA}$$

$$n = \frac{6 \times 10^{23}}{7 \text{ cm}^3} = 0.86 \times 10^{23} / \text{m}^3$$

$$v_d = \frac{1.1}{0.86 \times 1.6 \times 10^{-19} \times 78.5 \times 10^{-4}}$$

(volume of 63g Cu)

$$v_d = 0.01 \text{ cm/s.}$$

$$= 0.1 \text{ mm/s}$$

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CURRENT ELECTRICITY

JEE ADVANCED SYLLABUS

1. *Ohm's law*
2. *Series and parallel arrangements of resistances and cells*
3. *Kirchhoff's laws and applications to networks*
4. *charging and discharging of capacitor*
5. *Heating effect of current*

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

- A. Write Question Number (QN) which you are unable to solve at your own in **column A**.
- B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.
- C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		

Revision Strategy :

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Step-3: Solve Questions of Column B

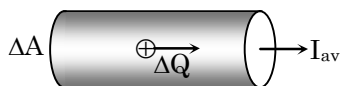
Step-4: Solve questions from other Question Bank, Problem book etc.

Current Electricity

KEY CONCEPT

1. Electric Current

It is the time rate of flow of charge through a conductor when there is net transfer of charge (say ΔQ) across a cross section during a time interval (say Δt), we define average electric current as



$$I_{av} = \frac{\Delta Q}{\Delta t}$$

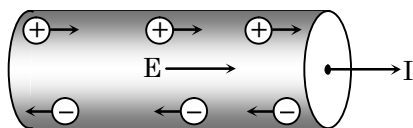
1.1 The instantaneous current at any instant

't' as
$$I_{in} = \frac{dq}{dt}$$

1.2 Current is one of the seven fundamental quantities. The S.I. unit of current is ampere.

$$1 \text{ Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ Second}}$$

1.3 The conventional direction of current is along the direction of flow of positive charge and opposite to motion of negative charge.



1.4 To generate electric current in a material it must have charge carriers and source of energy (or emf).

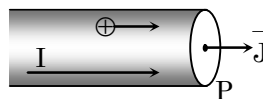
Type of material	Charge Carriers
Metal	Free electrons
Semiconductors	Free electrons & holes
Gas / Electrolyte	+ve and -ve ions

2. Current Density

2.1 The current density at a point is defined as a vector having magnitude equal to current per unit area surrounding that point and normal to the direction of charge flow. It is represented by \vec{J} .

2.2 At a point P if current I passes normally through area dA (see figure), then

$$\vec{J} = \frac{d\vec{I}}{dA}$$



2.3 The direction of current density is the direction of motion of positive charge at the point P.

2.4 Note that the area ΔA is normal to the current ΔI . If ΔA is not normal to I , but makes angle θ with the normal then

$$\vec{J} = \frac{dI}{\text{normal component of area}} = \frac{dI}{dA \cos \theta}$$

$$\Rightarrow dI = J dA \cos \theta$$

$$I = \int \vec{J} \cdot d\vec{A}$$

2.5 Its unit is ampere / meter² and dimension is $[M^0 L^{-2} T^0 A]$.

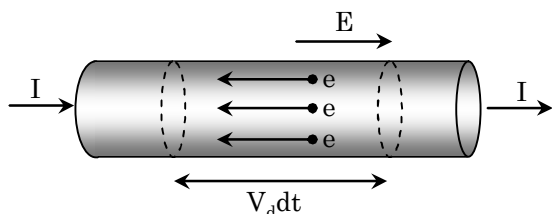
2.6 If a source of emf produces electric field inside a conductor of resistivity ρ , then current density at any point inside the conductor is

$$\vec{J} = \frac{\vec{E}}{\rho} = \sigma \vec{E} \left(\sigma = \frac{1}{\rho} = \text{conductivity} \right).$$

3. Relation between I and v_d

Consider any cylindrical element of a conductor of area of cross section A . If its free electrons are drifting with velocity v_d , the length of the element swept in time dt will be $v_d dt$. If the conductor has n free electrons per unit volume the number of electron in this volume will be $n(Av_d dt)$.

All these electrons cross the area A in time dt . Thus charge crossing in time dt across time area A is $dQ = n(Av_d dt)e$



or
$$I = \frac{dQ}{dt}$$

$$I = neAv_d \text{ and } J = \frac{I}{A} = nev_d$$

4. Mobility

Drift velocity acquired by a charge carrier (free electrons holes, ions etc.) per unit electric field is defined as its mobility. It is denoted by μ .

$$\mu = \frac{v_d}{E} = \frac{e\tau}{m} \quad [\because v_d = \frac{eE}{m}\tau]$$

4.1 It is measured in $= \frac{m/s}{\text{volt/m}} = \frac{(\text{meter})^2}{\text{volt} \times \text{sec}}$

4.2 Its dimension is $\mu = \frac{(AT)T}{M} = M^{-1} T^2 A^1$

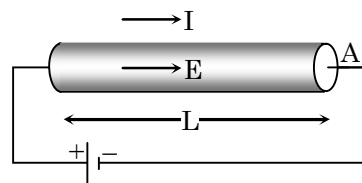
5. Ohm's Law

If there is no change in the physical state (such as temperature etc) of a conductor, magnitude of current is directly proportional to applied potential difference.

i.e.
$$V \propto I$$

$$V = RI$$

Where R is a constant called 'Electrical Resistance' of the conductor



resistance of a given conductor is

$$R = \left(\frac{m}{ne^2\tau} \right) \frac{L}{A}$$

or
$$R = \rho \frac{L}{A}$$

Where $\rho = \text{resistivity} = \frac{m}{ne^2\tau}$

$m \rightarrow$ mass of electrons,

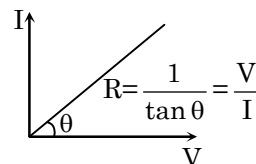
$e \rightarrow$ charge of electrons

$n \rightarrow$ no. of free electrons per unit volume,

$L \rightarrow$ Length of the conductor

$A \rightarrow$ Area of cross section perpendicular to current flow

5.1 A conducting device obeys ohm's law only if its physical state remains unchanged. In this case the graph between I and V is a straight line passing through origin ($\because V \propto I$). Such a conductor (e.g. metals, alloys etc) is called ohmic-conductor having resistance (R) equal to reciprocal of slope of I - V . curve (see figure)



5.2 For non ohmic resistors where linear relationship between current and voltage is not followed, we define **dynamic resistance** (r) as ratio of change in voltage to change in current at a given voltage. This is too measured in ohm.

$$r = \frac{\Delta V}{\Delta I}$$

5.3 If the length of wire is changed by stretching $\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$ as volume remains unchanged ($\ell_1 A_1 = \ell_2 A_2$)

5.4 If a conductor is stretched such that its radius is reduced to $1/n^{\text{th}}$ of its original value, its new resistance will be increased n^4 times.

5.5 If wire is stretched to increase its length by $x\%$ (& $x < 5\%$) then its resistance increases by $2x\%$.

5.6 Temperature dependence of resistance and resistivity

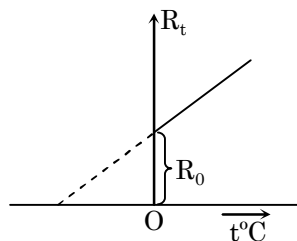
(a) Resistance as well as resistivity changes with change in temperature. This variation can be explained on the basis of temperature coefficient of resistance (or resistivity).

(b) The temperature coefficient of resistance is defined as fractional change in resistance per unit rise of temperature. It is denoted by α and is measured in $^{\circ}\text{C}^{-1}$ or K^{-1} .

$$\alpha = \frac{\Delta R}{R} \times \frac{1}{\Delta \theta}, \text{ therefore}$$

$$\Delta R = R \alpha \Delta \theta.$$

If R_t and R_0 are resistance of a conductor at $t^{\circ}\text{C}$ and 0°C respectively then



$$R_t - R_0 = R_0 \alpha (t - 0)$$

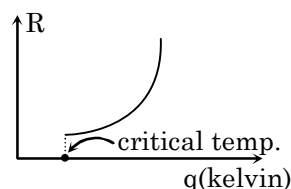
$$R_t = R_0 [1 + \alpha t]$$

where α = coefficient at 0°C .

If temperature coefficient of resistance at $t_1^{\circ}\text{C}$ is α and resistance is R_1 then resistance at any other temp. t_2 will be.

$$R_2 = R_1 [1 + \alpha (t_2 - t_1)]$$

(c) The general relationship can be represented graphically as :



The graph shows that at critical temperature the resistivity absolutely falls to zero. This phenomenon is known as **superconductivity**.

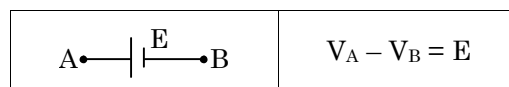
6. Cell terminology

6.1 Electromotive force : (EMF) :

It refers to the work done by the cell in moving unit + ve charge in the whole circuit including the cell once. So $E = \frac{W}{q}$

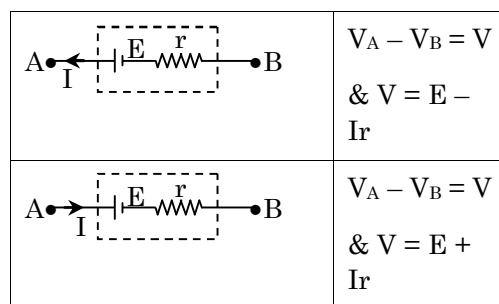
(It is measured in joule / coulomb or volt.)

In simple language it is potential difference across the terminals of cell when no current is drawn from it.



6.2 Internal resistance (r) :

The internal resistance of an ideal cell is zero. But for real cell it is non-zero and is always in series with the cell.



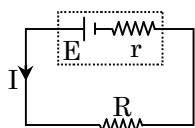
7. Power Transfer

It is defined as the time rate of energy transferred by a cell to the load. It is given by

$$W = q V$$

$$W = I^2 R t \quad [q = It, \text{ \& } V = IR]$$

therefore $\frac{W}{t} = P = I^2 r$



$$\Rightarrow P = \frac{E^2 R}{(R + r)^2} \quad \left[\because I = \frac{E}{R + r} \right].$$

From the equations it is clear that power transferred to the load will be maximum when

$$\frac{dP}{dR} = 0; \quad \frac{d}{dR} \left[\frac{E^2 R}{(R + r)^2} \right] = 0$$

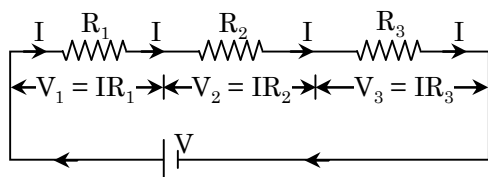
on solving it we get

$$R = r.$$

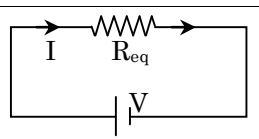
This shows that power transferred by a cell to the load will be maximum when external load is equal to internal resistance of the cell, i.e. $R = r$. This rule is known as **maximum power transfer theorem** and the power transferred to the load (R) in this condition is

$$P_{\max} = \frac{E^2 R}{(R + r)^2} = \frac{E^2}{4r}$$

8. Resistances in 'Series'



The equivalent circuit of the above figure is as



From the figure it is clear that

$$I = I_1 = I_2 = I_3 = \dots; \quad \text{and}$$

$$V_1 = IR_1, V_2 = IR_2 \text{ \& } V_3 = IR_3 \dots; \text{ and } V = V_1 + V_2 + V_3 + \dots$$

If equivalent resistance is R_{eq} then

$$V = IR_{eq}$$

$$\text{Therefore } R_{eq} = R_1 + R_2 + R_3 + \dots$$

- Since in series combination potential is divided according to $V \propto R$, hence is case of three resistors in series

$$V_1 : V_2 : V_3 = R_1 : R_2 : R_3 \text{ and } V = V_1 + V_2 + V_3$$

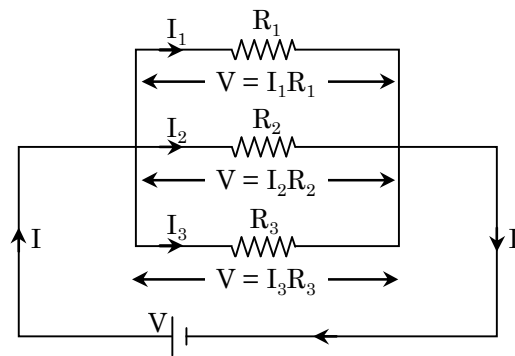
$$\text{i.e. } V_1 = \frac{R_1}{R_1 + R_2 + R_3} V$$

$$\text{and } V_2 = \frac{R_2}{R_1 + R_2 + R_3} V$$

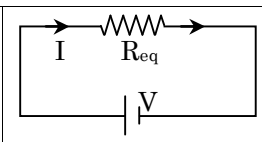
$$\text{and } V_3 = \frac{R_3}{R_1 + R_2 + R_3} V$$

9. Resistances in parallel

In parallel combination potential difference across each resistance is same, but total current from the source is divided in the inverse proportion of the resistance $\left(I \propto \frac{1}{R} \right)$.



The equivalent circuit of the above figure is as



From the figure we can say that

$$V = V_1 = V_2 = V_3 = \dots; \text{ and}$$

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}; \dots \text{ and}$$

$$I = I_1 + I_2 + I_3 + \dots$$

If equivalent resistance is R_P then

$$I = \frac{V}{R_{eq}}, \text{ therefore}$$

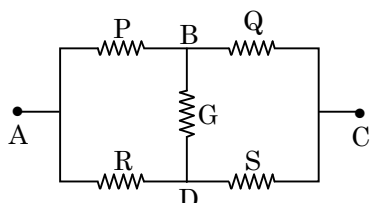
$$\frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

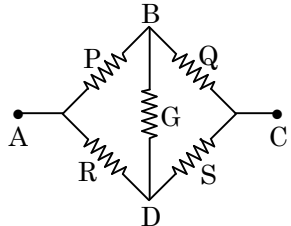
10. Wheatstone bridge type circuits

If in a network, resistance are arranged as in the circuit shown below, the network is called **Wheatstone bridge** and is said to be balanced

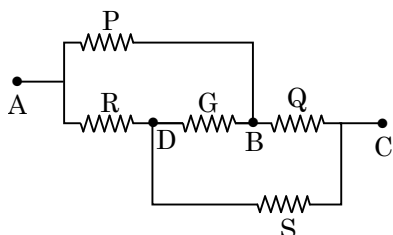
$$\text{If } \frac{P}{Q} = \frac{R}{S}$$



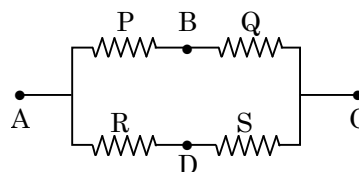
or



or



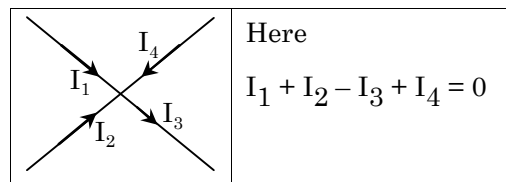
In a balanced Wheatstone bridge certain points are at same potential (see points B and D) and so no current flows through the resistor connected between such points (as the resistor G in above figures.) Therefore removing such resistors will have no effect on net resistance. Thus in a balanced Wheatstone bridge equivalent circuit resistance can be drawn as follows :



11. Kirchhoff's Laws

11.1 Kirchhoff's first law (junction law) :

It states that the algebraic sum of currents coming at a junction is equal to zero. $\Sigma I = 0$. This law is based on the principle of conservation of charges.



11.2 Kirchhoff's second law (loop law) :

It states that in an electric circuit the sum of the potential drop across different components is equal to zero. This is based on conservation of energy.

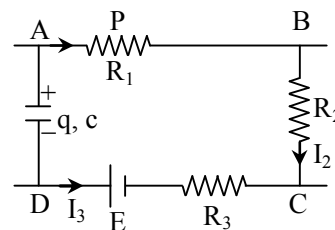
Sign convention

$$\begin{array}{c} \text{A} \xrightarrow{I} \text{---} \text{R} \text{---} \text{B} \\ V_A - V_B = +IR \end{array}$$

$$\begin{array}{c} \text{A} \text{---} \text{E} \text{---} \text{B} \\ V_A - V_B = +E \end{array}$$

$$\begin{array}{c} \text{A} \xrightarrow{q, C} \text{---} \text{B} \\ V_A - V_B = + \frac{q}{c} \end{array}$$

(If one moves from B to A the drop will be negative) Here we can apply the Kirchhoff's second law in the loop ABCDA and get.



$$+ I_1 R_1 + I_2 R_2 - I_3 R_3 - E - \frac{q}{c} = 0$$

- 11.3** The following points must be taken into consideration in applying the Kirchhoff's Law.
- (a) Assume the direction of current in each branch.
 - (b) In showing the currents in different branches, use Kirchhoff's junction law.
 - (c) Select the loops such a way that each new loop must have at least one new branch not considered previously.
 - (d) The number of loops must be equal to the number of variables.

12. Joule Heating

- 12.1** When current pass through a resistor, heat is generated at the rate of $I^2 R_0$ per unit time.
- 12.2** If current through the resistor is constant, heat generated in time t is $H = I^2 R t$
- 12.3** If current through the resistor is variable, heat generated in time t is

$$H = \int_0^t I^2 R \, dt$$

13. Electrical Instruments

13.1 Bulbs

- (a) Bulbs are rated to consume power at a given voltage. For example 100W, 220 Volt.
- (b) Power consumption is a variable quantity whereas its resistance is definite.
- (c) Resistance, $R = \frac{V^2}{P}$.
- (d) When bulbs with rating P_1, P_2, P_3 at same voltage V are connected in parallel and to a voltage V , they will consume power as specified.

$$P = P_1 + P_2 + P_3 + \dots$$

- (e) When bulbs with rating P_1, P_2, P_3 at same voltage V are connected in series, power consumed by all the bulbs are less than specified. Net power consumption P is given by

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \dots$$

- (f) When the circuit is complex, replace the bulb by pure resistance and solve the circuit.

13.2 Galvanometer

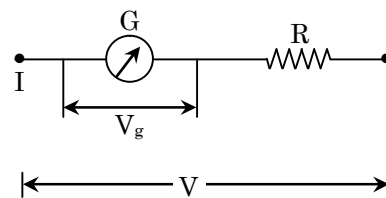
- (a) Galvanometer is an instrument which is sensitive to current. Whenever current flows through it, it gives deflection, and hence one can measure the current.
- (b) It can be converted to a voltage meter (voltmeter) or an ampere meter (ammeter) by making suitable modification.
- (c) **Conversion of Galvanometer to volt meter or to enhance the range of voltmeter :**

Let, V_g = Potential difference applied across the voltmeter to give full scale deflection.

G = Galvanometer resistance.

V = The desired range of the voltage to be measured.

R = Resistance connected in series to the voltmeter to achieve the requirement.

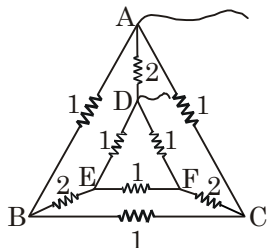


R is given by

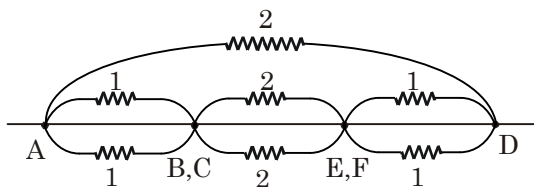
$$R = G \left(\frac{V}{V_g} - 1 \right)$$

SOLVED EXAMPLES

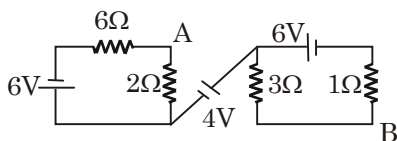
Ex.1 A network of nine conductors connects six points A, B, C, D, E and F as shown. The figures denote resistances in ohms. The equivalent resistance between A and D is



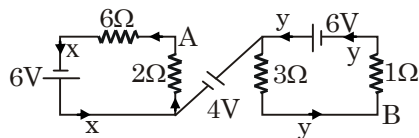
Sol. B and c are equipotential points and so are E and F. Here the circuit can be redrawn as shown in Figure. 1Ω and 1Ω in parallel sum up to $1/2W$; $2W$ and $2W$ in parallel sum up to $1W$; $1/2W$, $1W$, $1/2W$ in series sum up to $1/2 + 1 + 1/2 = 2\Omega$; 2Ω and 2Ω in parallel sum up to $= 1\Omega$.



Ex.2 In the network shown in the figure below, calculate the potential difference between A and B.



Sol. The distribution of current is shown in Fig., keeping in view that the inflow and outflow of current in a cell must be the same. Applying the loop rule to the left and right loops.



$$2x + 6x = 6$$

or

$$x = 0.5 \text{ A}$$

$$\text{and } 3y + 1y = 6$$

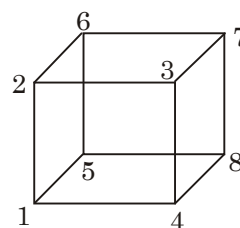
$$\text{or } y = 1.5 \text{ A}$$

$$V_{AB} = \sum r - \sum e = (-2 \times 0.5 + 3 \times 1.5) - 4 = -0.5 \text{ V}$$

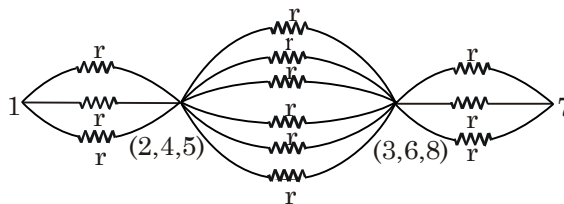
The minus sign shows that B is at a higher potential than A. Thus

$$V_{BA} = -1 \text{ V}$$

Ex.3 Find the resistance of a wire frame shaped as a cube when measured between points. 1 and 7 The resistance of each edge is r.



Sol. Symmetry about entrance point 1 and exit point 7 shows that 2, 4, 5 are equipotential points and 5, 6, and 8 are equipotential points. Hence the circuit can be redrawn as shown in Figure. The resistance r, r and r in parallel sum up to $r/3$.



r, r, r, r, r, r in parallel sum up to $r/6$ and r, r, r in parallel sum up to $r/3$. Next $r/3, r/6, r/3$ in series sum up to $5r/6$.

Ex.4 Two resistors with temperature coefficients of resistance α_1 and α_2 have resistances R_{01} and R_{02} at 0°C . Find the temperature coefficient of the compound resistor consisting of the two resistors connected in parallel.

$$\text{Sol. } R_1 = R_{01} (1 + \alpha_1 t)$$

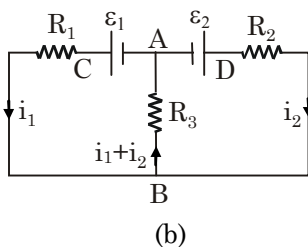
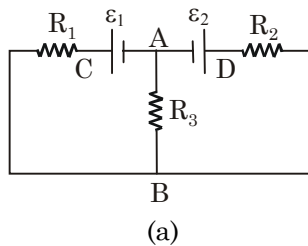
$$\text{and } R_2 = R_{02} (1 + \alpha_2 t)$$

Also
$$R = \frac{R_1 R_2}{R_1 + R_2} = R_0 (1 + \alpha t)$$

and
$$R_0 = \frac{R_{01} R_{02}}{R_{01} + R_{02}}$$

$$\begin{aligned} \therefore \frac{R_{01} R_{02}}{R_{01} + R_{02}} (1 + \alpha t) &= \frac{R_{01} R_{02} (1 + \alpha_1 t)(1 + \alpha_2 t)}{R_{01} + R_{02} + (R_{01} \alpha_1 + R_{02} \alpha_2) t} \\ \Rightarrow (1 + \alpha t) &= \frac{(1 + (\alpha_1 + \alpha_2) t)}{1 + \frac{R_{01} \alpha_1 + R_{02} \alpha_2}{R_{01} + R_{02}} t} \\ &= (1 + (\alpha_1 + \alpha_2) t) \left(1 - \frac{R_{01} \alpha_1 + R_{02} \alpha_2}{R_{01} + R_{02}} t \right) \\ \Rightarrow 1 &= \alpha t \\ &= 1 + 1 + (\alpha_1 + \alpha_2) t - \frac{R_{01} \alpha_1 + R_{02} \alpha_2}{R_{01} + R_{02}} t \\ &= 1 + \frac{R_{01} \alpha_1 + R_{02} \alpha_1 + R_{01} \alpha_2 + R_{02} \alpha_2 - R_{01} \alpha_1 - R_{02} \alpha_2}{R_{01} + R_{02}} t \\ &= 1 + \frac{R_{02} \alpha_1 + R_{01} \alpha_2}{R_{01} + R_{02}} \times t \\ \alpha &= \frac{R_{02} \alpha_1 + R_{01} \alpha_2}{R_{01} + R_{02}} \end{aligned}$$

Ex.5 Find the currents going through the three resistors R_1 , R_2 and R_3 in the circuit of figure.



Sol. Let us take the potential of the point. A to be zero. The potential at C will be ε_1 and that at D will be ε_2 . Let the potential at B

be V. The currents through the three resistors are i_1 , i_2 and $i_1 + i_2$ as shown in figure. Note that the current directed towards B equals the current directed away from B.

Applying Ohm's law to the three resistors R_1 , R_2 and R_3 we get

$$\varepsilon_1 - V = R_1 i_1 \quad \dots(i)$$

$$\varepsilon_2 - V = R_2 i_2 \quad \dots(ii)$$

$$\text{and} \quad V - 0 = R_3 (i_1 + i_2) \quad \dots(iii)$$

Adding (i) and (iii),

$$\begin{aligned} \varepsilon_1 &= R_1 i_1 + R_3 (i_1 + i_2) \\ &= (R_1 + R_3) i_1 + R_3 i_2 \quad \dots(iv) \end{aligned}$$

and adding (ii) and (iii),

$$\begin{aligned} \varepsilon_2 &= R_2 i_2 + R_3 (i_1 + i_2) \\ &= (R_2 + R_3) i_2 + R_3 i_1 \quad \dots(v) \end{aligned}$$

Equations (iv) and (v) may be directly written from Kirchhoff's loop law applied to the left half and the right half of the circuit.

Multiply (iv) by $(R_2 + R_3)$, (v) by R_3 and subtract to eliminate i_2 . This gives

$$\begin{aligned} i_1 &= \frac{\varepsilon_1 (R_2 + R_3) - \varepsilon_2 R_3}{(R_1 + R_3)(R_2 + R_3) - R_3^2} \\ i_1 &= \frac{\varepsilon_1 (R_2 + R_3) - \varepsilon_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \end{aligned}$$

Similarly eliminating i_1 from (iv) and (v) we obtain,

$$i_2 = \frac{\varepsilon_2 (R_1 + R_3) - \varepsilon_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

And so,

$$i_1 + i_2 = \frac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

Ex.6 A capacitor is connected to a 12 V battery through a resistance of 10 Ω . It is found that the potential difference across the capacitor rises to 4.0 V in 1 μ s. Find the capacitance of the capacitor (given $\ln(3/4) = 0.405$).

Sol. The charge on the capacitor during charging is given by

$$Q = Q_0 (1 - e^{-t/RC})$$

Hence, the potential difference across the capacitor is

$$V = Q/C = Q_0 / C (1 - e^{-t/RC})$$

Here at $t = 1 \mu s$, the potential difference is 4 V whereas the steady state potential difference is

$$Q_0 / C = 12 \text{ V So,}$$

$$4V = 12 \text{ V } (1 - e^{-t/RC})$$

$$\text{or, } 1 - e^{-t/RC} = \frac{1}{3}$$

$$\text{or, } e^{-t/RC} = \frac{2}{3}$$

$$\text{or, } \frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0.405$$

or,

$$RC = \frac{t}{0.405} = \frac{1 \mu s}{0.405} = 2.469 \mu s$$

$$\text{or, } C = \frac{2.469 \mu s}{10 \Omega} = 0.25 \mu F$$

Ex.7 All the edges of a block with parallel faces are unequal. Its longest edge is twice its shortest edge. The ratio of the maximum to minimum resistance between parallel faces is

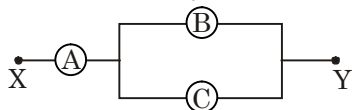
Sol. Let the edges be 2ℓ , a and ℓ , in decreasing order.

$$R_{\max} = \rho \frac{2\ell}{a\ell}$$

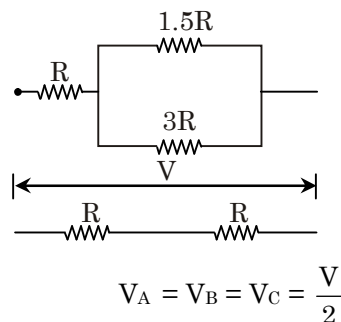
$$R_{\min} = \rho \frac{\ell}{2\ell a} = \frac{\rho}{2a}$$

$$\frac{R_{\max}}{R_{\min}} = 4$$

Ex.8 A, B and C are voltmeters of resistances R , $1.5 R$ and $3 R$ respectively. When some potential difference say V is applied between X and Y, the voltmeter readings are V_A , V_B and V_C respectively. What are the values of V_A , V_B & V_C ?

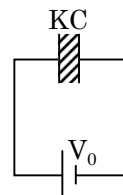


Sol.

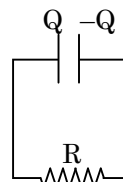


Ex.9 A parallel-plate capacitor of capacitance C , filled with a dielectric of dielectric constant k , and charged to a potential V_0 . It is now disconnected from the cell and the slab is removed. If it now discharges, with time constant τ , through a resistance, the potential difference across it will be V_0 after what time.

Sol.



$$Q = KCV_0$$



$$q = Qe^{-t/\tau}$$

$$\Rightarrow \frac{q}{C} = \frac{Q}{C} e^{-t/\tau}$$

$$V_0 = \frac{KCV_0}{C} e^{-t/\tau}$$

$$\Rightarrow V_0 = KV_0 e^{-t/\tau}$$

$$\frac{1}{K} = e^{-t/\tau}$$

$$\text{or } \log K = \frac{t}{\tau} \\ t = \tau \log_e K$$

EXERCISE (Level-1)

Questions
based on

Current & Resistance definitio:

- Q.1** In copper, each copper atom releases one electron. If a current of 1.1 A is flowing in the copper wire of uniform cross-sectional area of diameter 1 mm, then drift velocity of electrons will approximately be- (Density of copper = $9 \times 10^3 \text{ kg/m}^3$, Atomic weight of copper = 63)
(A) 10.3 mm/s (B) 0.1 mm/s
(C) 0.2 mm/s (D) 0.2 cm/s
- Q.2** A current of 5A exists in a 10Ω resistance for 4 minutes. The number of electrons and charge in coulombs passing through any section of the resistor in this time are -
(A) 75×10^{20} , 600C
(B) 75×10^{21} , 600C
(C) 75×10^{20} , 1200C
(D) 75×10^{19} , 1200C
- Q.3** A potential difference V exists between the ends of a metal wire of length ℓ . The drift velocity will be doubled if -
(A) V is doubled
(B) ℓ is doubled
(C) The diameter of the wire is doubled
(D) The temperature of the wire is doubled
- Q.4** The current in a conductor varies with time t is $I = 2t + 3t^2$ where I is in ampere and t in seconds. Electric charge flowing through a section of conductor during $t = 2 \text{ sec}$ to $t = 3 \text{ sec}$ is -
(A) 10 C (B) 24 C (C) 33 C (D) 44 C
- Q.5** The current in a copper wire is increased by increasing the potential difference between its end. Which one of the following statements regarding n, the number of charge carriers per unit volume in the wire and v the drift velocity of the charge carriers is correct -
(A) n is unaltered but v is decreased
(B) n is unaltered but v is increased
(C) n is increased but v is decreased
(D) n is increased but v is unaltered

Q.6

Consider two conducting wires of same length and material, one wire is solid with radius r. The other is a hollow tube of outer radius 2r while inner r. The ratio of resistance of the two wires will be -

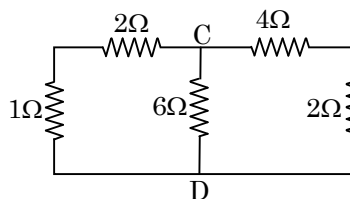
- (A) 1 : 1 (B) 1 : 2
(C) 3 : 1 (D) 1 : 4

Questions
based on

Series & parallel combination of Resistance

Q.7

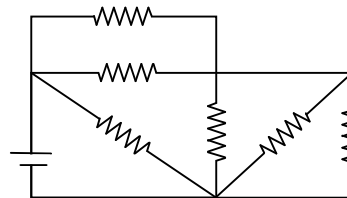
Equivalent resistance between point C and D in the combination of resistance shown is -



- (A) 3 Ω (B) 1 Ω (C) 1.5 Ω (D) 0.5 Ω

Q.8

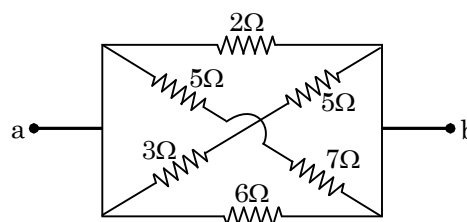
In the figure shown each resistor is of 20Ω and the cell has emf 10 volt with negligible internal resistance. Then rate of joule heating in the circuit is (in watts) -



- (A) 100/11 (B) 10000/11
(C) 11 (D) None of these

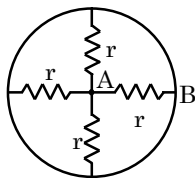
Q.9

Find the equivalent resistance between a & b



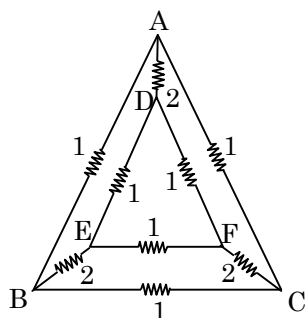
- (A) $\frac{7}{8} \Omega$ (B) $\frac{8}{7} \Omega$ (C) $\frac{6}{7} \Omega$ (D) $\frac{7}{6} \Omega$

- Q.10** The equivalent resistance between point A and B is -



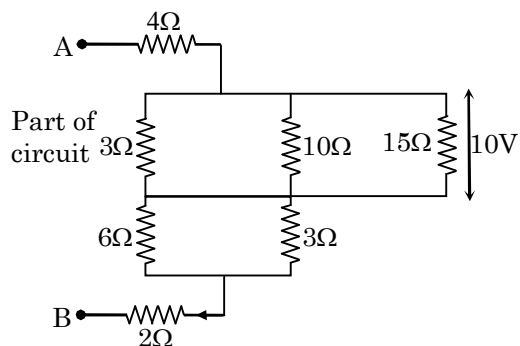
- (A) $4r$ (B) $2r$ (C) r (D) $\frac{r}{4}$

- Q.11** A network of nine conductors connects six points A, B, C, D, E and F as shown below. The digits denote resistances in Ω . Find the equivalent resistance between B and C-



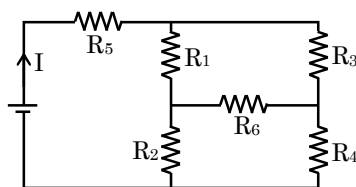
- (A) $\frac{2}{15} \Omega$ (B) $\frac{7}{12} \Omega$
(C) $\frac{5}{12} \Omega$ (D) $\frac{11}{12} \Omega$

- Q.12** Calculate the potential difference between points A and B and current flowing through the 10Ω resistor in the part of the network below -



- (A) 20 V, 2A (B) 50 V, 1A
(C) 40 V, 1A (D) 30 V, 1A

- Q.13** In the given circuit, it is observed that the current I is independent of the value of the resistance R_6 . Then the resistance values must satisfy -

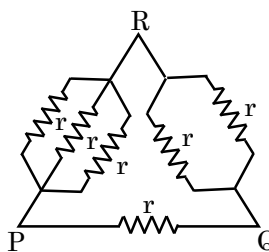


- (A) $R_1 R_2 R_5 = R_3 R_4 R_6$
(B) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$
(C) $R_1 R_4 = R_2 R_3$
(D) $R_1 R_3 = R_2 R_4 = R_5 R_6$

- Q.14** Two wires of equal diameters of resistivities ρ_1 and ρ_2 and lengths x_1 and x_2 are joined in series. The equivalent resistivity of the combination is -

- (A) $\frac{\rho_1 x_1 + \rho_2 x_2}{x_1 + x_2}$ (B) $\frac{\rho_1 x_2 - \rho_2 x_1}{x_1 - x_2}$
(C) $\frac{\rho_1 x_2 + \rho_2 x_1}{x_1 + x_2}$ (D) $\frac{\rho_1 x_1 + \rho_2 x_2}{\rho_1 + \rho_2}$

- Q.15** In the circuit shown in the figure, equivalent resistance is maximum -



- (A) Between P & Q
(B) Between P & R
(C) Between R & P
(D) Same between all the points

- Q.16** The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be -
- (A) 100% (B) 50%
(C) 300% (D) 200%

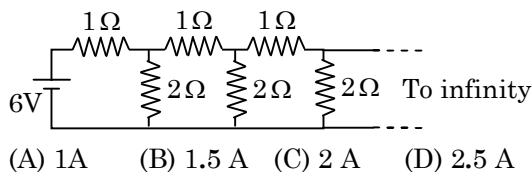
EXERCISE (Level-2)

Single correct answer type questions

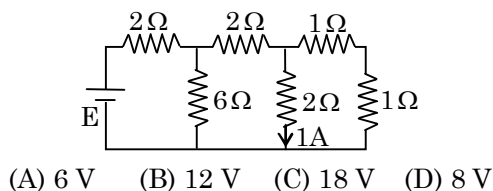
- Q.1** A carbon and an aluminium wire connected in series. If the combination has resistance of 30 ohm at 0°C , what is the resistance of carbon and aluminium wire at 0°C so that the resistance of the combination does not change with temperature - [$\alpha_c = -0.5 \times 10^{-3} (\text{C}^\circ)^{-1}$ and $\alpha_{Al} = 4 \times 10^{-3} (\text{C}^\circ)^{-1}$]

(A) $\frac{10}{3} \Omega, \frac{80}{3} \Omega$ (B) $\frac{80}{3} \Omega, \frac{10}{3} \Omega$
(C) 10 Ω , 80 Ω (D) 80 Ω , 10 Ω

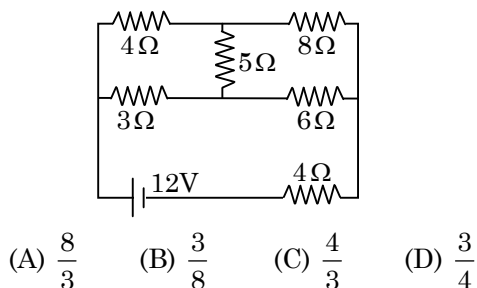
- Q.2** An infinite ladder network of resistance is constructed with 1 Ω and 2 Ω resistance. The 6V battery between A and B has negligible internal resistance. The current that passes through 2 Ω resistance nearest to the battery is -



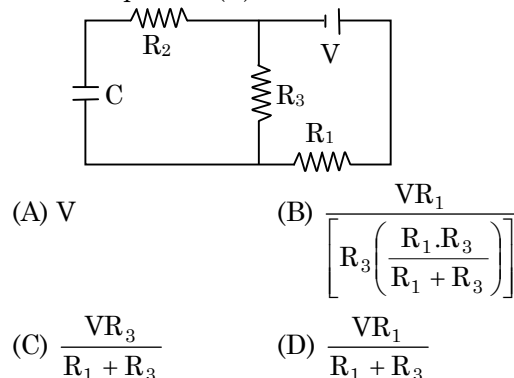
- Q.3** The emf of the battery shown in the figure is given by -



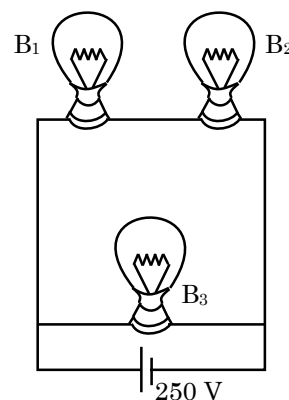
- Q.4** In the given figure the ratio of current in 8 Ω and 3 Ω will be -



- Q.5** In figure the steady state voltage drop across capacitor (C) is -

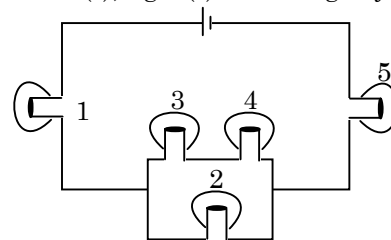


- Q.6** A 100 W bulb B_1 , and two 60 W bulbs B_2 and B_3 , are connected to a 250 V source, as shown in the figure. Now W_1 , W_2 and W_3 are the output powers of the bulbs B_1 , B_2 and B_3 , respectively. (Rated potential of each bulb is 250 V) select correct alternative -



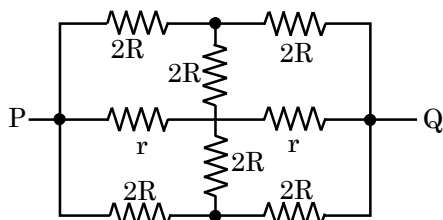
(A) $W_1 > W_2 = W_3$ (B) $W_1 > W_2 > W_3$
(C) $W_1 < W_2 = W_3$ (D) $W_1 < W_2 < W_3$

- Q.7** In the fig below the bulbs are identical, which bulb(s), light(s) most brightly ?



(A) 1 only (B) 4 only
(C) 2 and 3 (D) 1 and 5

- Q.8** The effective resistance between points P and Q of the electrical circuit shown in the figure is -

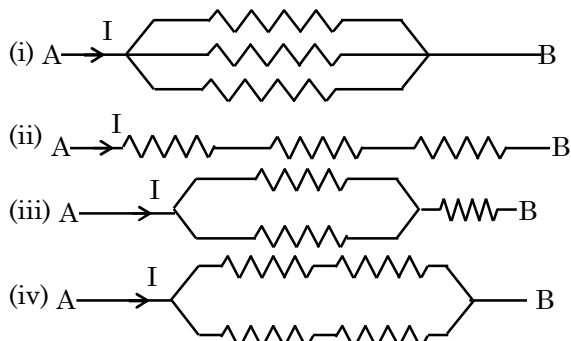


- (A) $2Rr / (R + r)$
 (B) $8R(R + r) / (3R + r)$
 (C) $2r + 4R$
 (D) $5R / 2 + 2r$

- Q.9** A long resistance wire is divided into $2n$ parts. Then n parts are connected in series and the other n parts in parallel separately. Both combinations are connected to identical supplies. Then the ratio of heat produced in series to parallel combinations will be -
 (A) $1 : 1$ (B) $1 : n^2$ (C) $1 : n^4$ (D) $n^2 : 1$

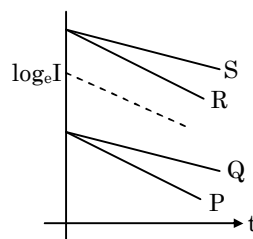
- Q.10** In a potentiometer experiment it is found that no current pass through the galvanometer when the terminals of the cell are connected across 125 cm of potentiometer wire. On shunting the cell by a 2Ω resistance the balancing length reduces to half. The internal resistance of the cell is -
 (A) 4Ω (B) 2Ω (C) 1Ω (D) 0.5Ω

- Q.11** Arrange the order of power dissipated in the given circuits, if the same current is passing through all the circuits. The resistance of each resistor is 'r' -



- (A) $P_2 > P_3 > P_4 > P_1$ (B) $P_1 > P_4 > P_2 > P_3$
 (C) $P_3 > P_1 > P_4 > P_2$ (D) $P_2 > P_4 > P_1 > P_3$

- Q.12** A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of $\log_e I$ with respect to time. If the resistance is changed to $2x$, the new graph will be -



- (A) P (B) Q (C) R (D) S

- Q.13** The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of battery whose internal resistance is 0.5Ω . If the balance point is obtained at $\ell = 30$ cm from the positive end the e.m.f. of the battery is -

- (A) $\frac{30E}{(100 - 0.5)}$
 (B) $\frac{30(E - 0.5i)}{(100)}$, where i is the current in the potentiometer wire
 (C) $\frac{30E}{100}$
 (D) $\frac{30E}{100.5}$

- Q.14** An ammeter reads upto 1 ampere. Its internal resistance is 0.81 ohm . To increase the range to 10A the value of the required shunt is -

- (A) 0.3Ω (B) 0.9Ω
 (C) 0.09Ω (D) 0.03Ω

- Q.15** The total current supplied to the circuit by the battery is -

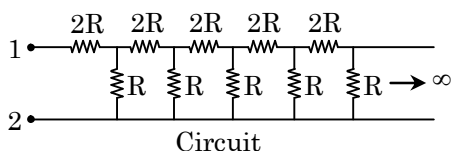
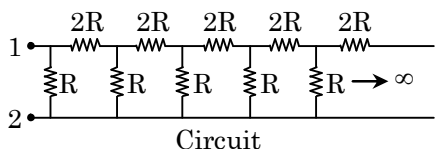


- (A) 1 A (B) 2 A (C) 4 A (D) 6 A

EXERCISE (Level-3)

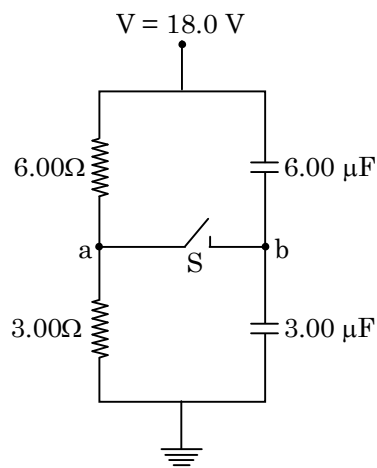
Part-A : Multiple correct answer type questions

- Q.1** Two circuits (as shown in figure) are called circuit A and circuit B. The equivalent resistance of circuit A is x and that of circuit B is y between 1 and 2.



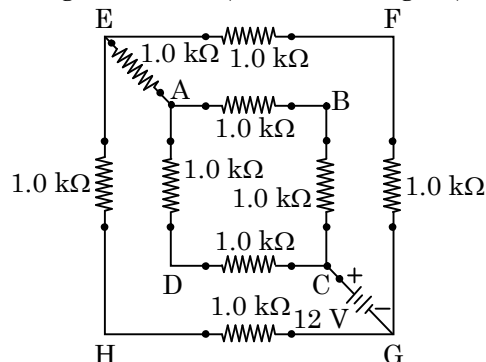
- (A) $y > x$ (B) $y = (\sqrt{3} + 1) R$
 (C) $xy = 2R^2$ (D) $y - x = 2R$

- Q.2** Study the following circuit diagram in figure and mark the correct options.



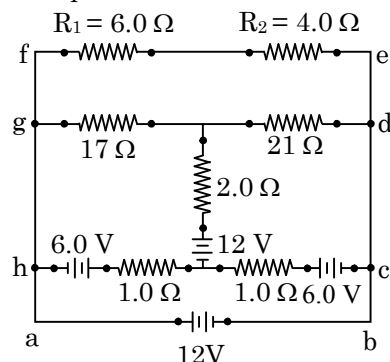
- (A) The potential of point a with respect to point b in the figure when switch S is open is $-6V$.
 (B) The points a and b are at the same potential, when S is opened.
 (C) The charge flowing through switch S when it is closed is $54 \mu C$.
 (D) The final potential of b with respect to ground when switch S is closed is $8 V$.

- Q.3** In the given circuit (as shown in figure)



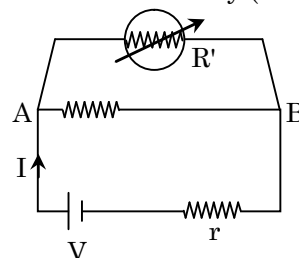
- (A) the equivalent resistance between C and G is $3 k\Omega$.
 (B) the current provided by the source is 4 mA
 (C) the current provided by the source is 8 mA
 (D) voltage across points G and E is $4 V$

- Q.4** In the circuit shown in figure, mark the correct option.



- (A) potential drop across R_1 is $3.2 V$
 (B) Potential drop across R_2 is $5.4 V$
 (C) Potential drop across R_1 is $7.2 V$
 (D) Potential drop across R_2 is $4.8 V$

- Q.5** Consider a simple circuit shown in figure stands for a variable resistance R' . R' can vary from R_0 to infinity, r is internal resistance of the battery ($r \ll R \ll R'$).



- (A) Potential drop across, AB is nearly constant as R' is varied
 (B) Current through R' is nearly a constant as R' is varied
 (C) Current I depends sensitively on R'
 (D) $I \geq \frac{V}{r + R}$ always

Q.6 When no current is passed through a conductor—

- (A) the free electrons do not move
 (B) the average speed of free electrons over a large period of time is zero
 (C) the average velocity of free electrons over a large period of time is zero
 (D) the average of the velocities of all the free electrons at an instant is zero

Q.7 A current passes through a wire of non-uniform cross-section. Which of the following quantities are independent of the cross-section —

- (A) the charge crossing in a given time interval
 (B) drift velocity
 (C) current density
 (D) free-electron density

Q.8 Two resistors having equal resistances are joined in series and a current is passed through the combination. Neglect any variation of resistance as a temperature change. In a given time interval—

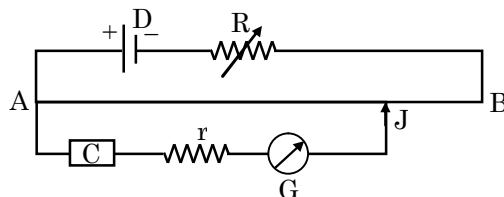
- (A) equal amounts of thermal energy must be produced in the resistors
 (B) unequal amounts of thermal energy may be produced
 (C) the temperature must rise equally in the resistors
 (D) the temperature may rise equally in the resistors

Q.9 Two fuse wire of rating 10 A and 20 A are connected in different type. Then —

- (A) In parallel combination works as a fuse of 30 A
 (B) In parallel combination works as a fuse of 10 A
 (C) In series combination works as a fuse of 10 A
 (D) In series combination works as a fuse of 20 A

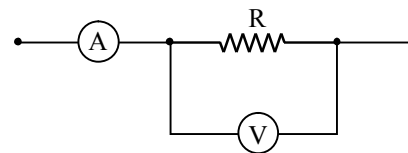
Q.10 In a potentiometer wire experiment the emf of a battery in the primary circuit is 20 Volt and its internal resistance is $5\ \Omega$. There is a resistance box (in series with the battery and the potentiometer wire) whose resistance can be varied from $120\ \Omega$ to $170\ \Omega$. Resistance of the potentiometer wire is $75\ \Omega$. The following potential difference can be measured using this potentiometer—
 (A) 5V (B) 6V (C) 7V (D) 8V

Q.11 In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0 . C is a cell of internal resistance r . The galvanometer G does not give zero deflection for any position of the jockey J. Which of the following cannot be a reason for this ?



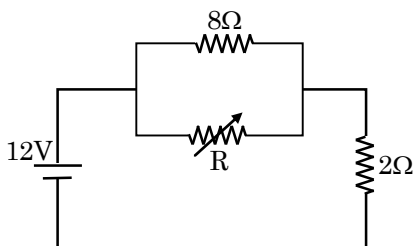
- (A) $r > R_0$
 (B) $R \gg R_0$
 (C) emf of C > emf of D
 (D) The negative terminal of C is connected to A

Q.12 In the circuit shown the readings of ammeter and voltmeter are 4 A and 20 V respectively. The meters are non ideal, then R is —



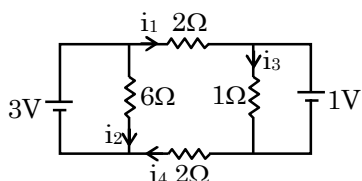
- (A) $5\ \Omega$ (B) less than $5\ \Omega$
 (C) greater than $5\ \Omega$ (D) between $4\ \Omega$ & $5\ \Omega$

- Q.13** The value of the resistance R in figure is adjusted such that power dissipated in the 2Ω resistor is maximum. Under this condition –



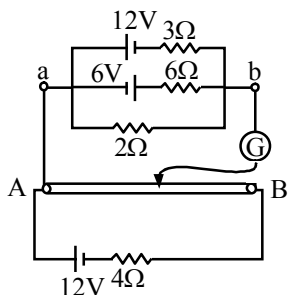
- (A) $R = 0$
 (B) $R = 8\Omega$
 (C) Power dissipated in the 2Ω resistor is 72 W
 (D) Power dissipated in the 2Ω resistor is 8 W

- Q.14** In the circuit shown, current in different branches are marked. Select the correct alternatives -



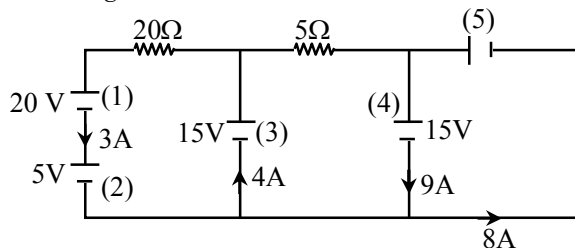
- (A) $i_1 = \frac{1}{2}\text{ A}$ (B) $i_4 = \frac{1}{2}\text{ A}$
 (C) $i_2 = \frac{1}{2}\text{ A}$ (D) $i_3 = 1\text{ A}$

- Q.15** In a potentiometer circuit, a uniform wire of 10 m having resistance 20Ω is fixed between A and B as shown in fig. Neglecting resistance of connecting wires, select the correct options



- (A) distance of null point from A is 5 m .
 (B) distance of null point from A is 3 m
 (C) at null point, current through 4Ω is 0.5 A
 (D) at null point, current through 3Ω resistor is 3 A

- Q.16** In the given networks, the batteries getting charged are

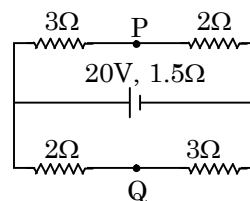


- (A) 1 & 3 (B) 1, 3 & 5
 (C) 1 & 4 (D) 1, 2 & 5

- Q.17** The emf of a cell is :

- (A) the potential difference across its terminals
 (B) the potential difference across its terminals when no current is passing through it
 (C) the heat produced when the cell is connected across a one ohm resistance
 (D) the total work done per coulomb of electricity taken in a circuit in which the cell is connected

- Q.18** In the given circuit :



- (A) the current through the battery is 5.0 amp
 (B) P and Q are at the same potential
 (C) P is 2.5 V higher than Q
 (D) Q is 2.5 V higher than P

Part-B : Assertion Reason type Questions

The following questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

- (A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
 (B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.
 (C) If Assertion is true but the Reason is false.
 (D) If Assertion is false but Reason is true.

Q.19 Assertion : If the length of the conductor is doubled, the drift velocity will become half of the original value (keeping potential difference unchanged).

Reason : At constant potential difference, drift velocity is inversely proportional to the length of the conductor.

Q.20 Assertion : In a chain of bulbs, 50 bulbs are joined in series. One bulb is fused now. If the remains 49 bulbs are again connected in series across the same supply then light gets increased in the room.

Reason : The resistance of 49 bulbs will be less than 50 bulbs.

Q.21 Assertion : Current is passed through a metallic wire, heating it red. When cold water is poured on half of its portion, then rest of the half portion becomes more hot.

Reason : Resistance decreases due to decrease in temperature and then current through wire increases.

Q.22 Assertion: A domestic electrical appliance, working on a three pin, will continue working even if the top pin is removed.

Reason : The third pin is used only as a safety device.

Q.23 Assertion : In parallel combination of electrical appliances, in home circuit total power consumption is equal to the sum of the rated powers of the individual appliances.

Reason : In parallel combination, in home circuit the voltage across each appliance is the same, as required for the proper working of electrical appliance.

Q.24 Assertion : When the cell is in the open circuit there is no force on a test charge inside the electrolyte of the cell.

Reason : There is no electric field inside the cell, when the cell is in open circuit.

Q.25 Assertion : The emf of the driver cell in the potentiometer experiment should be greater than the e.m.f. of the cell to be determined.

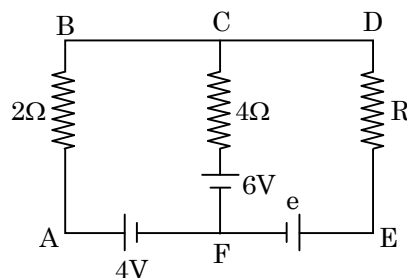
Reason : The fall of potential across the potentiometer wire should not be less than the e.m.f. of the cell to be determined.

Q.26 Assertion : A potentiometer of longer length is used for accurate measurement.

Reason : The potential gradient for a potentiometer of longer length with a given source of e.m.f. becomes small.

Part-C : Column Matching type Questions

Q.27 A circuit is shown in figure R is a nonzero variable but finite resistance. e is some unknown emf with polarities as shown. Match the columns.



Column-I

- (A) Current passing through 4Ω resistance can be zero.
- (B) Current passing through 4Ω resistance can be from F to C.
- (C) Current passing through 4Ω resistance can be from C to F.
- (D) Current passing through 2Ω resistance will be from B to A.

Column-II

- (P) possible if $e = 6V$
- (Q) possible if $e > 6V$
- (R) possible if $e < 6V$
- (S) possible for any value of e from zero to infinity

EXERCISE (Level-4)

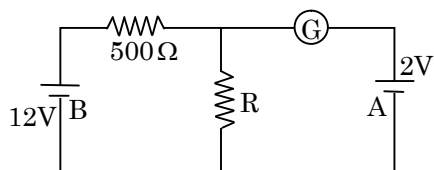
Old Examination Questions

Section-A [JEE Main]

Q.1 A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be - **[AIEEE-2005]**

(A) 10^3 (B) 10^5 (C) 99995 (D) 9995

Q.2 In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be- **[AIEEE-2005]**



(A) 200 Ω (B) 100 Ω (C) 500 Ω (D) 1000 Ω

Q.3 Two sources of equal emf are connected to an external resistance R. The internal resistances of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero, then - **[AIEEE-2005]**

(A) $R = R_2 \times (R_1 + R_2) / (R_2 - R_1)$

(B) $R = R_2 - R_1$

(C) $R = R_1 R_2 / (R_1 + R_2)$

(D) $R = R_1 R_2 / (R_2 - R_1)$

Q.4 An energy source will supply a constant current into the load if its internal resistance is - **[AIEEE-2005]**

(A) equal to the resistance of the load

(B) very large as compared to the load resistance

(C) zero

(D) non-zero but less than the resistance of the load

Q.5 In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω , the balancing length becomes 120 cm. The internal resistance of the cell is -

[AIEEE-2005]

(A) 1 Ω (B) 0.5 Ω (C) 4 Ω (D) 2 Ω

Q.6 A material 'B' has twice the specific resistance of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. Then for the two wires to have the same resistance, the ratio ℓ_B / ℓ_A of their respective lengths must be - **[AIEEE-2006]**

(A) $\frac{1}{4}$ (B) 2 (C) 1 (D) $\frac{1}{2}$

Q.7 The Kirchhoff's first law ($\sum i = 0$) and second law ($\sum iR = \sum E$), where the symbols have usual meanings, are respectively based on - **[AIEEE-2006]**

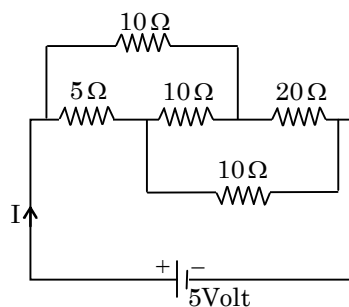
(A) conservation of momentum, conservation of charge

(B) conservation of charge, conservation of energy

(C) conservation of charge, conservation of momentum

(D) conservation of energy, conservation of charge

Q.8 The current I drawn from the 5 volt source will be - **[AIEEE-2006]**



(A) 0.67 A

(B) 0.17 A

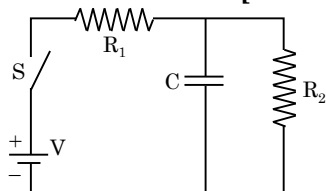
(C) 0.33 A

(D) 0.5 A

Section-B [JEE Advanced]

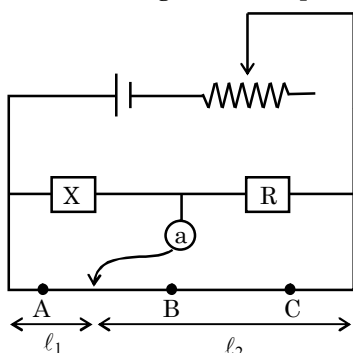
- Q.1** In the given circuit the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0(1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters as shown in the circuit.

[IIT-JEE 2005]



- Q.2** An unknown resistance is to be determined using resistance R_1 , R_2 , and R_3 . If their corresponding null points are A , B and C . Which of the following will give most accurate reading?

[IIT-JEE 2005]



- Q.3** A galvanometer having Resistance 100Ω is used to form an ammeter with the help of resistance 0.1Ω . The maximum deflection of galvanometer is at $100 \mu A$. Find the smallest current when Galvanometer shows maximum deflection-

[IIT-JEE 2005]

- (A) 100.1 mA (B) 1000.1 mA
(C) 10.01 mA (D) 1.001 mA

- Q.4** A $4 \mu F$ capacitor, a resistance of $2.5 \text{ M}\Omega$ is in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor

[Given $\ln(2) = 0.693$] [IIT-JEE 2005]

- (A) 13.86 s (B) 6.93 s
(C) 7 s (D) 14 s

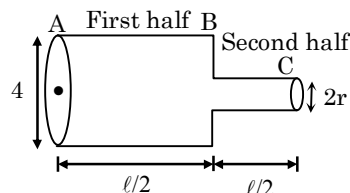
- Q.5** An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100Ω resistor carrying current 1 A for 5 minutes supplies heat to the gas. The change in internal energy of the gas is -

[IIT-JEE 2005]

- (A) 10 kJ (B) 30 kJ (C) 20 kJ (D) 0 kJ

- Q.6** Consider a cylindrical element as shown in the figure. Current flowing through the element is I and resistivity of material of the cylinder is ρ . Choose the correct option out the following -

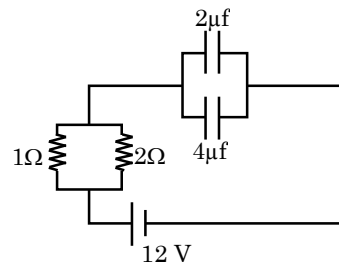
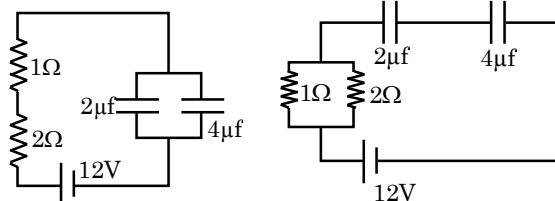
[IIT-JEE 2006]



- (A) Power loss in second half is four times the power loss in first half
(B) Voltage drop in first half is twice of voltage drop in second half
(C) Current density in both halves are equal
(D) Electric field in both halves is equal

- Q.7** Time constant for the given circuits are -

[IIT-JEE 2006]



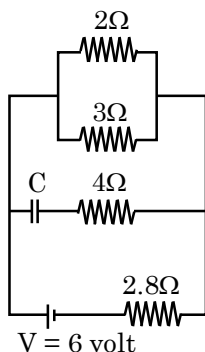
- (A) $18 \mu s$, $\frac{8}{9} \mu s$, $4 \mu s$ (B) $18 \mu s$, $4 \mu s$, $\frac{8}{9} \mu s$
(C) $4 \mu s$, $\frac{8}{9} \mu s$, $18 \mu s$ (D) $\frac{8}{9} \mu s$, $18 \mu s$, $4 \mu s$

EXERCISE (Level-5)

Review Exercise

- Q.1** Calculate the steady state current in the 2Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is $0.2\mu\text{F}$.

[IIT-JEE 1982]



- Q.2** Two resistors, 400Ω , and 800Ω are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10Ω resistance is used for this purpose. What will be the reading in the ammeter? Similarly, if a voltmeter of 1000Ω resistance is used to measure the potential difference across the 400Ω resistor, what will be the reading in the voltmeter?

[IIT-JEE 1982]

- Q.3** A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current the temperature of the wire is raised by ΔT in a time t . A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t . The value of N is –

[IIT-JEE 2001]

- (A) 4 (B) 6 (C) 8 (D) 9

- Q.4** A piece of copper and another of germanium are cooled from room temperature to 80K . The resistance of –

[IIT-JEE 1988]

- (A) Each of them increases
(B) Each of them decreases
(C) Copper increases and germanium decreases
(D) Copper decreases and germanium increases

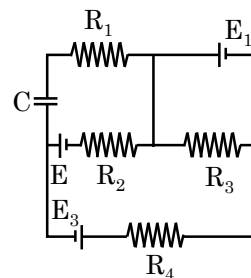
- Q.5** In the given circuit –

$$E_1 = 3, E_2 = 2, E_3 = 6 \text{ volt,}$$

$$R_1 = 2\Omega, R_4 = 6 \text{ ohm, } R_3 = 2\Omega,$$

$$R_2 = 4 \text{ ohm, } C = 5\mu\text{F}.$$

Find the current in R_3 and the energy stored in the capacitor. [IIT-JEE 1998]



- Q.6** A micro ammeter has a resistance of 100Ω and full scale range of $50\mu\text{A}$. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combinations –

MCQ [IIT-JEE 1991]

- (A) 50 V range with $10\text{ K}\Omega$ resistance in series
(B) 10 V range with $200\text{ K}\Omega$ resistance in series
(C) 5 mA range with 1Ω resistance in parallel
(D) 10 mA range with 1Ω resistance in parallel

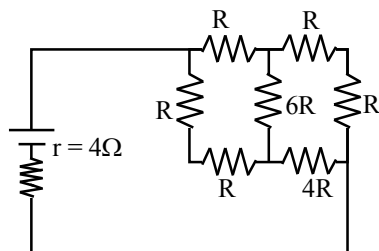
Q.7 Read the following statements carefully—
 Y : The resistivity of semiconductor decreases with increases of temperature.

Z : In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature.

Select the correct statement (s) from the following [IIT-JEE 1993]

- (A) Y is true but Z is false
- (B) Y is false but Z is true
- (C) Both Y and Z are true
- (D) Y is true and Z is the correct reason for Y

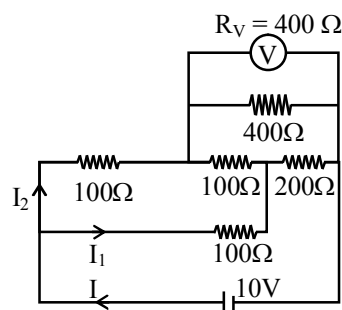
Q.8 A battery of internal resistance $4\ \Omega$ is connected to the network of resistance as shown. In order that maximum power can be delivered to the network, the value of R in ohm should be - [IIT-JEE 1995]



- (A) $\frac{4}{9}$ (B) 2 (C) $\frac{8}{3}$ (D) 18

Q.9 A uniform copper wire of mass $2.23 \times 10^{-3}\text{ kg}$ carries a current of 1 A when 1.7 V is applied across it. Calculate its length and area of cross-section. If the wire is uniformly stretched to double its length, calculate the new resistance. Density of copper is $8.92 \times 10^3\text{ kg m}^{-3}$ and resistivity is $1.7 \times 10^{-8}\ \Omega\text{m}$. [Roorkee 95]

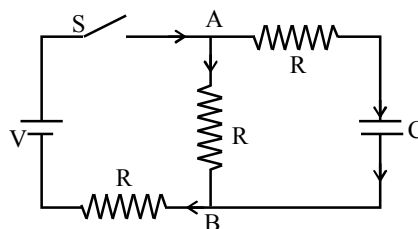
Q.10 An electrical circuit is shown in figure. Calculate the potential difference across the resistor of 400 ohm, as will be measured by the voltmeter V of resistance 400 ohm, either by applying Kirchhoff's rules or otherwise. [IIT-JEE 1996]



Q.11 In the circuit shown in Fig., the battery is an ideal one, with emf V . The capacitor is initially uncharged. The switch S is closed at time $t = 0$.

- (a) Find the charge Q on the capacitor at time t .
- (b) Find the current in AB at time t . What is its limiting value at $t \rightarrow \infty$?

[IIT-JEE 1997]



Q.12 A steady current flows in a metallic conductor of non-uniform cross section. The quantity/quantities constant along the length of the conductor is - [IIT-JEE 1997]

- (A) current, electric field and drift speed
- (B) drift speed only
- (C) current and drift speed
- (D) current only

Q.13 A series combination of $0.1\text{ M}\ \Omega$ resistor and a $10\ \mu\text{F}$ capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged up to 0.75 V is approximately (in seconds) [IIT-JEE 1997]

- (A) ∞ (B) $\log_e 2$
- (C) $\log_{10} 2$ (D) Zero

ANSWER KEY

EXERCISE (Level-1)

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (B) | 2. (C) | 3. (A) | 4. (B) | 5. (B) | 6. (C) | 7. (C) |
| 8. (C) | 9. (B) | 10. (D) | 11. (B) | 12. (B) | 13. (C) | 14. (A) |
| 15. (A) | 16. (C) | | | | | |

EXERCISE (Level-2)

- | | | | | | | |
|---------|--------|---------|---------|---------|---------|---------|
| 1. (B) | 2. (B) | 3. (B) | 4. (D) | 5. (C) | 6. (D) | 7. (D) |
| 8. (A) | 9. (B) | 10. (B) | 11. (A) | 12. (B) | 13. (C) | 14. (C) |
| 15. (C) | | | | | | |

EXERCISE (Level-3)

Part-A

- | | | | | | | |
|--------------|----------|-------------|-----------|----------|-----------|---------------|
| 1. (A,B,C,D) | 2. (A,C) | 3. (A,B,D) | 4. (C,D) | 5. (A,D) | 6. (C,D) | 7. (A,D) |
| 8. (A,D) | 9. (A,C) | 10. (A,B,C) | 11. (A) | 12. (C) | 13. (A,C) | 14. (A,B,C,D) |
| 15. (B,C,D) | 16. (C) | 17. (B,D) | 18. (A,D) | | | |

Part-B

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 19. (A) | 20. (B) | 21. (A) | 22. (A) | 23. (A) | 24. (C) | 25. (A) |
| 26. (A) | | | | | | |

Part-C

27. $A \rightarrow Q$; $B \rightarrow P,Q,R$; $C \rightarrow Q$; $D \rightarrow P,Q,R,S$

EXERCISE (Level-4)

SECTION-A

- | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|
| 1. (D) | 2. (B) | 3. (B) | 4. (B) | 5. (D) | 6. (B) | 7. (B) |
| 8. (D) | | | | | | |

SECTION-B

- | | |
|---|--|
| 1. $Q_0 = \frac{CVR_2}{R_1 + R_2}$ and $\alpha = \frac{R_1 + R_2}{R_1 R_2 C}$ | 2. The null point at B will give most accurate reading |
| 3. (A) | 4. (A) |
| 5. (B) | 6. (A) |
| | 7. (A) |

EXERCISE (Level-5)

- | | | | |
|---|-------------------------------------|---------|--------|
| 1. 0.9 A | 2. 4.96 mA, 1.58 volt | 3. (B) | 4. (D) |
| 5. 1.5 A from right to left and energy stored is $1.44 \times 10^{-5} \text{ J}$ | 6. (B, C) | 7. (C) | 8. (B) |
| 9. $\ell = 5\text{m}$, $A = 5 \times 10^{-8}$, $R_1 = 4 R = 6.8 \Omega$ | 10. p.d. = $\frac{20}{3} \text{ V}$ | | |
| 11. (a) $Q = \frac{VC}{2} \left(1 - e^{-2t/3RC}\right)$ (b) $I_1 = \frac{V}{2R} - \frac{Ve^{-\frac{2t}{3RC}}}{6R}$, $\lim_{t \rightarrow \infty} I_1 = \frac{V}{2R}$ | 12. (D) | 13. (B) | |

ATOMIC STRUCTURE

JEE ADVANCED SYLLABUS

1. *Rutherford's model*
2. *Bohr's model*
3. *Quantum numbers*
4. *Electronic configuration of elements (upto atomic number 36) Aufbau principle*
5. *Pauli's exclusion principle and Hund's rule*
6. *Spectrum of hydrogen atom*
7. *de-Broglie relations*
8. *Uncertainty principle*
9. *Quantum mechanical model*
10. *Shapes of s, p and d-orbitals*

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

- A. Write Question Number (QN) which you are unable to solve at your own in **column A**.
- B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.
- C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Topic wise practice questions		
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of column A

Step-3: Solve Questions of Column B

Step-4: Solve questions from other Question Bank, Problem book etc.

Atomic Structure

KEY CONCEPT

1. Works Related with Scientists

Sr. No.	Particle	Symbol	Nature	Charge (in esu) $\times 10^{-10}$	Mass (in amu)	Discovered by
1.	Electron	e^-	–	4.8029	0.0005486	J.J. Thomson
2.	Proton	p^+	+	+ 4.8029	1.0072	Goldstein
3.	Neutron	n	0		1.0090	Chadwick
4.	Positron	e^+ , $1e^0$, β^+	+	+ 4.8029	0.0005486	Anderson (1932)
5.	Neutrino	ν	0	0	< 0.00002	Pauli
6.	Anti-proton	p^-	–	4.8029	1.00787	Chamberlain Sugri and Weighland (1955)
7.	Photon	$h\nu$	0	0	0	Planck
8.	Graviton	G	0	0	0	
9.	Positive (mu meson)	μ^+	+	+ 4.8029	0.1152	Yukawa (1935)
10.	Negative(mu meson)	μ^-	–	– 4.8029	0.1152	Yukawa (1935)
11.	Positive pi(meson)	π^+	+	+ 4.8029	0.1514	Powell (1947)
12.	Negative pi(meson)	π^-	–	– 4.8029	0.1514	
13.	Neutral pi (meson)	π^0	0	0	0.1454	

2. Rutherford's α -particle scattering experiment

$$N(\theta) \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \dots\dots\dots (i)$$

$$N(\theta) \propto \frac{1}{K.E.^2} \dots\dots\dots (ii)$$

$$N(\theta) \propto Z^2 \dots\dots\dots (iii)$$

$$N(\theta) \propto \frac{1}{r^2} \dots\dots\dots (iv)$$

Here,

Z = atomic number of element of metal foil

K.E. = K.E. of α -particle (initially)

θ = Scattering angle

r = Distance of screen from foil

3. Radius of Nucleus

$$R = R_0 A^{1/3} \dots\dots(i)$$

$$\text{or } R = 1.4 \times 10^{-15} A^{1/3} \text{ metre} \dots\dots(ii)$$

value of R_0 can be 1.1×10^{-15} to 1.44×10^{-15} metre.

4. Some Formulae Related with Bohr Model

$$(i) \quad \frac{mv^2}{r} = \frac{Ze^2}{r^2}$$

$$(ii) \quad mvr = \frac{nh}{2\pi}$$

$$(iii) \quad v = \frac{2\pi Ze^2}{nh} = 2.188 \times 10^8 Z/n \text{ cm/sec}$$

$$(iv) \quad r = \frac{n^2 h^2}{4\pi^2 m Z e^2} = 0.529 \times \left(\frac{n^2}{Z}\right) \text{ \AA}$$

$$(v) \quad E_T = \frac{-2\pi^2 m Z^2 e^4}{n^2 h^2} = -KE = \frac{PE}{2}$$

$$(vi) \quad E_T = \frac{-Ze^2}{2r} = K.E. + P.E.$$

$$(vii) \quad E_T = \frac{-2\pi^2 m K^2 Z^2 e^4}{n^2 h^2}$$

$$\begin{aligned} (viii) \quad E_T &= -21.8 \times 10^{-19} Z^2/n^2 \text{ J/atom} \\ &= -13.6 Z^2/n^2 \text{ eV/atom} \\ &= 21.8 \times 10^{-12} \frac{Z^2}{n^2} \text{ erg} \\ &= -1312 \text{ kJ/mol} \end{aligned}$$

$$(x) \quad E_1 < E_2 < E_3 < \dots\dots < E_\infty \quad (E_\infty = 0)$$

$$(xi) \quad (E_2 - E_1) > (E_3 - E_2) > (E_4 - E_3) > \dots\dots,$$

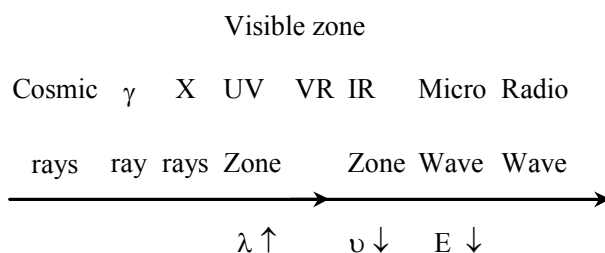
$$\begin{aligned} (xii) \quad \text{For H-atom :} \quad E_1 &= -13.6 \text{ eV} \\ E_2 &= -3.4 \text{ eV} \\ E_3 &= -1.5 \text{ eV} \\ E_4 &= -0.85 \text{ eV} \\ E_5 &= -0.54 \text{ eV} \end{aligned}$$

$$(xiii) \quad I.E. = 13.6 Z^2 \text{ eV/atom}$$

$$\begin{aligned} (xiv) \quad \text{Number of revolution per sec by an } e^- \\ = \frac{0.657 \times 10^{16} Z^2}{n^3} \end{aligned}$$

$$\begin{aligned} (xv) \quad \text{Time taken for one revolution} \\ = \frac{1.52 \times 10^{-16} n^3}{Z^2} \end{aligned}$$

5. Order of wave length & Frequency of electromagnetic radiation



6. Hydrogen Spectrum

(i) Various Series of spectrum lines :

Series of Lines	Transition	Spectrum Zone	Wave Length
1. Lyman	$n_2 = 2, 3, 4, \dots$ to $n_1 = 1$	Ultraviolet	$< 3800 \text{ \AA}$
2. Balmer	$n_2 = 3, 4, 5, \dots$ to $n_1 = 2$	Visible	$3800-7800 \text{ \AA}$
3. Paschen	$n_2 = 4, 5, 6, \dots$ to $n_1 = 3$	Infrared	$> 7800 \text{ \AA}$
4. Bracket	$n_2 = 5, 6, 7, \dots$ to $n_1 = 4$	"	"
5. Pfund	$n_2 = 6, 7, 8, \dots$ to $n_1 = 5$	"	"
6. Humphury	$n_2 = 7, 8, 9, \dots$ to $n_1 = 6$	"	"

(ii) Wave number ($\bar{\nu}$) and wavelength (λ) of spectral lines :

$$\bar{\nu} = \frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Here, $R_H = \frac{2\pi^2 m e^4}{ch^3}$ (R_H = Rydberg constant ;
 $= 109678 \text{ cm}^{-1}$)

(iii) Total no. of spectrum line = $\frac{n(n-1)}{2}$

7. In Sommer field Model

$$P_\phi = n_\phi \left(\frac{h}{2\pi} \right) \quad \& \quad P_r = n_r \left(\frac{h}{2\pi} \right)$$

& $P = P_r + P_\phi$ or $n = n_r + n_\phi$

$$\frac{\text{Semi major axis}}{\text{Semi minor axis}} = \frac{a}{b} = \frac{n}{\ell}$$

8. de-Broglie Concept

(i) $\lambda = \frac{h}{mv} = \frac{h}{p}$

(ii) $\frac{1}{2}mv^2 = eV$ (for electron)

(iii) $\lambda = \frac{h}{\sqrt{2emV}}$

(iv) $\lambda = \frac{12.25}{\sqrt{V}}$

(Here λ is in \AA , V is in volt) (for electron)

(v) $\lambda = \frac{h}{\sqrt{2mK.E.}}$

(vi) $\lambda = \frac{12.25}{\sqrt{K.E.}}$ (for electron)

(Here λ is in \AA and K.E. is in eV)

9. Heisenberg Uncertainty Principle

$$(\Delta x)(\Delta p) \geq \frac{h}{4\pi} \quad \& \quad (\Delta x)(\Delta v) \geq \frac{h}{4\pi m}$$

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi} \quad (\text{For energy and time})$$

$$\& \Delta \phi \cdot \Delta \theta \geq \frac{h}{4\pi} \quad (\text{For angular motions})$$

10. Shrodinger Theory

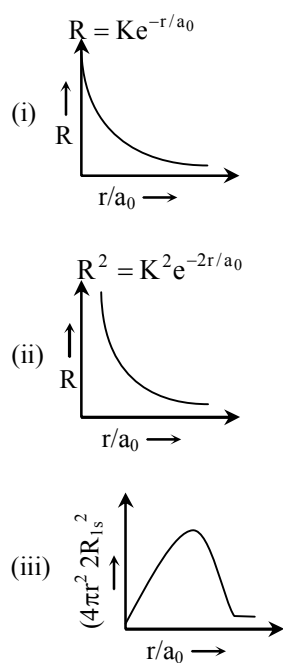
(a) Shrodinger wave equation :

(i) $\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \dots (i)$ or

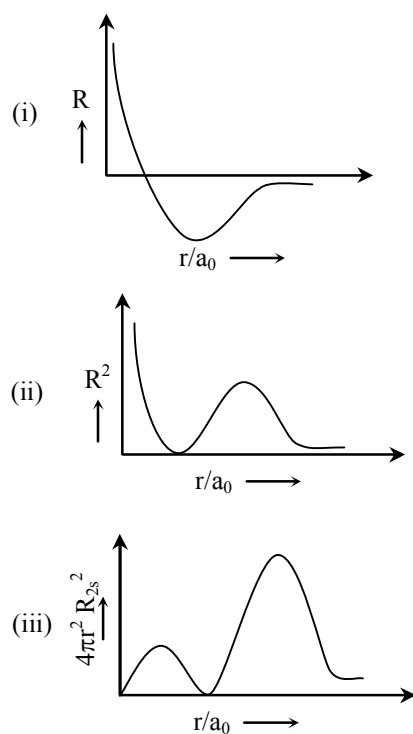
(ii) $E\psi = \hat{H}\psi \dots (ii)$

(b) Plots of radial probability distribution function ($4\pi r^2 R^2$)

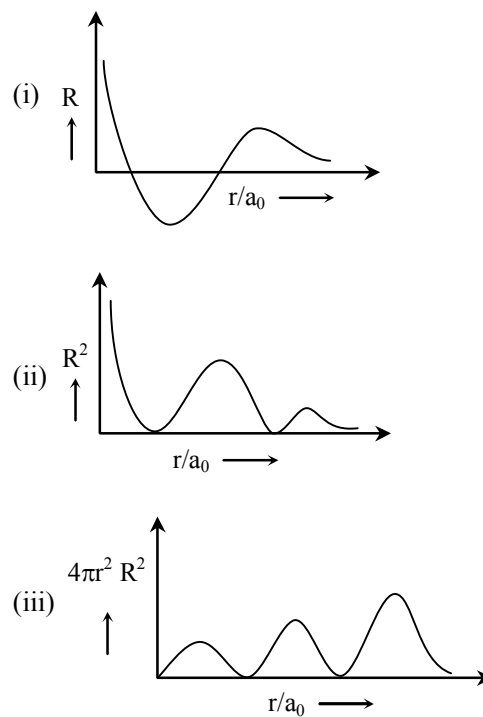
For 1s →



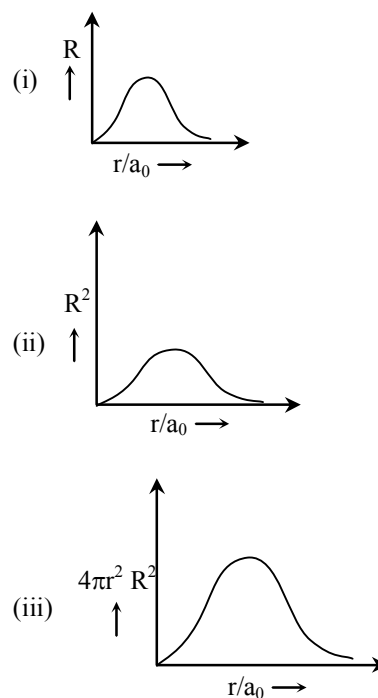
For 2s →



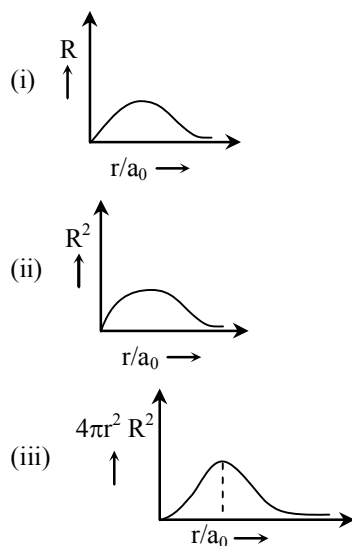
For 3s →



For 2p → $R = Kre^{-r/2a_0}$



For 3d →



11. Nodes

- (i) Radial nodes or spherical nodes = $n - \ell - 1$
- (ii) Angular nodes = ℓ
- (iii) Total nodes = $n - 1$

12. Quantum arithmetic

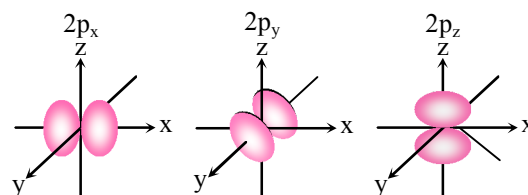
Sub shell	Value of ℓ	Value of m	No. of orbitals	Max. no. of e^-
s	0	0	1	2
p	1	0, ± 1	3	6
d	2	0, ± 1 , ± 2	5	10
f	3	0, ± 1 , ± 2 , ± 3	7	14

13. Shape of orbitals

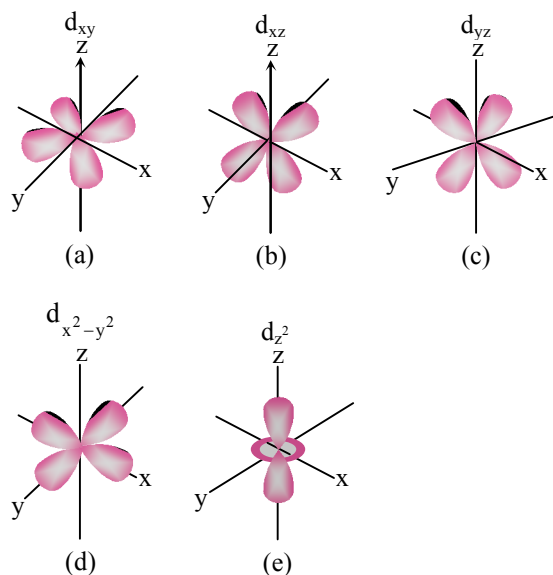
- (i) Boundary surface diagram for 1s orbital.



- (ii) Boundary surface diagrams of the three 2p orbitals.



- (iii) Boundary surface diagrams of the five 3d orbitals.



14. Some important points

- (a) No. of e^- s in any subshell = $2(2\ell + 1)$
- (b) No. of orbitals in any subshell = $(2\ell + 1)$
- (c) Orbital angular momentum of e^-
 $= \sqrt{[\ell(\ell+1)]} \hbar/2\pi$
- (d) Spin angular momentum of $e^- = \sqrt{[s(s+1)]} \hbar/2\pi$
- (e) No. of Max. e^- in any shell = $2n^2$
- (f) Max. number of orbitals in any shell = n^2
- (g) Max. number of subshell in any shell = n
- (h) Excited state of e^- is always equals to $(n-1)$
- (i) According to sommerfeld no. of elliptical orbit is equal to $(n-1)$

- (j) Total nodes = $(n - 1)$
- (k) Value of $\ell = 0$ to $(n - 1)$
- (l) Penultimate shell = $(n - 1)$
- (m) Spin multiplicity = $2S + 1$
(Here $\Rightarrow S = n/2$ and n = total no. of unpaired electrons)

15. Magnetic Moment

$$\text{Magnetic Moment} = \sqrt{n(n+2)} \text{ B.M.},$$

$$1 \text{ B.M.} = \frac{eh}{4\pi m_e}, = 9.27 \times 10^{-24} \text{ JT}^{-1};$$

n = total no. of unpaired e^-

16. Photo electric effect

$$h\nu = w + \frac{1}{2}mv^2 \quad \dots\dots\dots(i)$$

$$w = h\nu_0 \text{ (work function)} \quad \dots\dots\dots(ii)$$

17. Electron filling principles

(i) Aufbau Principle :

According to this principle, "In the ground state, the atomic orbitals are filled in order of increasing energies". i.e. in the ground state the electrons occupy the lowest orbitals available to them.

Order of filling of $e^- \rightarrow 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, \dots\dots\dots, 5f, 6d, 7p$.

(ii) Pauli's Exclusion Principle :

According to this principle, "No two electrons in an atom can have all the four quantum numbers n, ℓ, m and s identical".

(iii) Hund's Rule of Maximum Multiplicity :

According to this rule "Electron pairing will not take place in orbitals of same energy until all the available orbital of a given sub shell contain one electron each with parallel spin".

18. Isosters

Substance which have same number of electron and atoms called Isosters.

eg. CO_2	N_2O
22	22

SOLVED EXAMPLES

Ex.1 The ratio of the wave lengths of last lines of Balmer and Lyman series is -

- (A) 4 : 1 (B) 27 : 5
(C) 3 : 1 (D) 9 : 4

Sol.(A) The wave length of a spectral line may be given by the following expression

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Lyman series $n_1 = 1$, For Balmer series $n_1 = 2$

For the last line in both the series $n_2 = \infty$

For Lyman series

$$\frac{1}{\lambda_L} = R \left(1 - \frac{1}{\infty} \right) = R (1 - 0) = R$$

$$\lambda_L = \frac{1}{R}$$

For Balmer series

$$\frac{1}{\lambda_B} = R \left(\frac{1}{4} - \frac{1}{\infty} \right) = \frac{R}{4}$$

$$\lambda_B = \frac{4}{R}$$

$$\frac{\lambda_B}{\lambda_L} = \frac{4}{R} \times \frac{R}{1} = \frac{4}{1}$$

Ex.2 Consider an electron which is brought close to the nucleus of the atom from an infinite distance, the energy of the electron-nucleus system -

- (A) increases (B) decreases
(C) remains same (D) none of these

Sol.(B) The energy (P.E.) of the electron is a function of its distance from the nucleus and is given by Coulomb's law as

$$\text{P.E.} = - \frac{e^2}{r}$$

Total energy = K.E. + P.E.

$$= \frac{1}{2} mv^2 - \frac{e^2}{r} \left(\frac{1}{2} mv^2 = \frac{e^2}{2r} \right) = - \frac{e^2}{2r}$$

as 'r' decreases energy will go on decreasing.

Ex.3 Using arbitrary energy units we can calculate that 864 arbitrary units (a.u.) are required to transfer an electron in hydrogen atom from the most stable Bohr's orbit to the largest distance from the nucleus -

$$n = \infty \quad E = 0 \text{ Arbitrary units}$$

$$n = 4$$

$$n = 3$$

$$n = 2$$

$$n = 1 \quad E = -864 \text{ Arbitrary units}$$

The energy required to transfer the electron from third Bohr's orbit to the orbit $n = \infty$ will be -

- (A) 96 Arbitrary units
(B) 192 Arbitrary units
(C) 288 Arbitrary units
(D) 384 Arbitrary units

Sol.(A) The energy of first Bohr's orbit of H-atom

$$- \frac{2\pi^2 me^4}{h^2} = -864$$

The energy of third Bohr's orbit of H atom

$$= - \frac{2\pi^2 me^4}{h^2} \times \frac{1}{3^2} = -864 \times \frac{1}{9}$$

$$= -96 \text{ Arbitrary units}$$

Energy required to separate the electron

$$= E_{\infty} - E_n$$

$$= 0 - (-96)$$

$$= 96 \text{ Arbitrary units}$$

Ex.4 In an electronic transition, the wavelength of a spectral line is inversely related to -

- (A) The nuclear charge of the atom
(B) The difference in energy levels
(C) The velocity of electron
(D) The number of orbitals involved in transition

Sol.(A)
$$\frac{1}{\lambda} = \frac{2\pi^2 me^4 Z^2}{ch^3 (4\pi\epsilon_0)^2} = \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\lambda \propto \frac{1}{Z^2}$$

- Ex.5** The ratio of time periods in first and second orbits of hydrogen atom is -
 (A) 1 : 4 (B) 1 : 8
 (C) 1 : 2 (D) 2 : 1

Sol.(B) Time period in first orbit (T_1) = $\frac{2\pi r_1}{v_1}$

Time period in second orbit (T_2) = $\frac{2\pi r_2}{v_2}$

$$\frac{T_1}{T_2} = \frac{2\pi r_1}{v_1} \times \frac{v_2}{2\pi r_2}$$

$$= \frac{r_1}{r_2} \times \frac{v_2}{v_1}$$

Velocity of electron in first orbit

$$= \frac{2.188 \times 10^8}{1} \text{ cm s}^{-1}$$

Velocity of electron in second orbit

$$= \frac{2.188 \times 10^8}{2} \text{ cm s}^{-1}$$

Radius of first orbit

$$= 0.528 \times 10^{-8} \text{ cm}$$

Radius of second orbit

$$= 0.528 \times 10^{-8} \times 4 \text{ cm}$$

$$\frac{T_1}{T_2} = \frac{0.528 \times 10^{-8}}{0.528 \times 10^{-8} \times 4} \times \frac{2.188 \times 10^8}{2 \times 2.188 \times 10^8} = \frac{1}{8}$$

- Ex.6** In a hydrogen atom, the largest amount of energy will be required in -

- (A) $n_2 \rightarrow n_3$ transition (B) $n_\infty \rightarrow n_1$ transition
 (C) $n_1 \rightarrow n_2$ transition (D) $n_3 \rightarrow n_5$ transition

Sol.(C) $E_n = \frac{2\pi^2 me^4}{n^2 h^2 (4\pi\epsilon_0)^2}$ and

$$\Delta E = - \frac{2\pi^2 me^4}{h^2 (4\pi\epsilon_0)^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

- Ex.7** The ion that is isoelectronic with CO is -

- (A) CN^- (B) O_2^+
 (C) O_2^- (D) N_2^+

Sol.(A) $\text{CO } 6 + 8 = 14$

$$\text{CN}^- 6 + 7 + 1 = 14$$

$$\text{O}_2^+ 8 \times 2 - 1 = 15$$

$$\text{O}_2^- 8 \times 2 + 1 = 17$$

$$\text{N}_2^+ 7 \times 2 - 1 = 13$$

- Ex.8** Of the following, which of the statement(s) regarding Bohr theory is/are correct ?

- (A) Kinetic energy of an electron is half of the magnitude of its potential energy
 (B) Kinetic energy of an electron is negative of total energy of electron
 (C) Energy of electron decreases with increase in the value of principal quantum number
 (D) The ionization energy of H-atom in the first excited state is the negative of one fourth of the energy of an electron in the ground state

Sol.(A,B,D)

The energy of an electron in H-like atoms is given by

$$E = \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2} mv^2 - \frac{Ze^2}{(4\pi\epsilon_0)r}$$

From the stability of the circular motion of electron, we have

$$\frac{mv^2}{r} = \frac{Ze^2}{(4\pi\epsilon_0)r^2};$$

$$\text{Hence } E = \frac{1}{2} \frac{Ze^2}{(4\pi\epsilon_0)r} - \frac{Ze^2}{(4\pi\epsilon_0)r}$$

$$\text{or } E = - \frac{1}{2} \frac{Ze^2}{(4\pi\epsilon_0)r}$$

- Ex.9** An element with atomic mass Z consists of two isotopes of mass number $Z-1$ and $Z+2$. The percentage abundance of the heavier isotope is-

- (A) 0.25 (B) 33.3
 (C) 66.6 (D) 75

Sol.(B) $Z = \frac{(Z-1)x + (Z+2)(100-x)}{100}$

- Ex.10** Frequency ratio between violet (400 nm) and red (750 nm) radiations in the visible spectrum, is -

- (A) 8/15 (B) 4/15
 (C) 15/8 (D) None of these

Sol.(C) $v = \frac{c}{\lambda}$

For violet (400 nm) $v_1 = \frac{c}{400 \times 10^{-9}}$

For red (750 nm) $v_2 = \frac{c}{750 \times 10^{-9}}$

$$\frac{v_1}{v_2} = \frac{750}{400} = \frac{15}{8}$$

Ex.11 The orbital angular momentum of an electron in 2s orbital is -

(A) $+\frac{1}{2} \frac{h}{2\pi}$ (B) zero

(C) $\frac{h}{2\pi}$ (D) $\sqrt{2} \frac{h}{2\pi}$

Sol.(B) orbital angular momentum = $\sqrt{\ell(\ell+1)} \frac{h}{2\pi}$

$$\sqrt{\ell(\ell+1)} = h \text{ for S-orbital } \ell = 0$$

So orbital angular momentum is zero.

Ex.12 The sub-shell that comes after f-sub-shell is called g-sub-shell. The number of g-sub orbitals in g-sub-shell and the total number of orbitals in the principal orbital respectively are-

- (A) 10 and 25 (B) 9 and 25
(C) 11 and 23 (D) 15 and 45

Sol.(B) For a g-subshell : $n = 5$ and $\ell = 4$

$$\text{Number of orbitals g-subshell} = (2\ell + 1) = 9$$

$$\text{Total number of orbitals in atom} = n^2 = (5)^2 = 25$$

Ex.13 Two isotopes of Boron are found in the nature with atomic weights 10.01(I) and 11.01 (II). The atomic weight of natural Boron is 10.81. The percentage of (I) and (II) isotopes in it are respectively -

- (A) 20 and 80 (B) 10 and 90
(C) 15 and 75 (D) 30 and 70

Sol.(A) Let x% of I (10.01) is mixed with II (11.01) and the atomic weight become 10.81. Then

$$\frac{(10.01)x + (11.01)(100 - x)}{100} = 10.81$$

$$10.01x - 11.01 \times + 1101 = 10.81 \times 100$$

$$\text{or } -x = 1081 - 1101$$

$$\text{or } -x = -20$$

$$\therefore x = 20$$

$$\text{So ratio} = 20\% \text{ and } 100 - 20 = 80\%$$

Ex.14 For a d-electron, the orbital angular momentum is -

(A) $\sqrt{6} h/2\pi$ (B) $\sqrt{2} h/2\pi$

(C) $h/2\pi$ (D) $2.h/2\pi$

Sol.(A) Angular momentum L is given by

$$L = \sqrt{\ell(\ell+1)} \cdot \frac{h}{2\pi}$$

For d-orbital, $\ell = 2$, so

$$L = \sqrt{2(2+1)} \frac{h}{2\pi} = \sqrt{6} \frac{h}{2\pi}$$

Ex.15 The energy of the emitted photon when an electron in Be^{3+} ion returns from $n = 2$ level to ground state is -

- (A) $2.616 \times 10^{-17} \text{ J}$ (B) $26.16 \times 10^{-17} \text{ J}$
(C) $216.6 \times 10^{-17} \text{ J}$ (D) $2616 \times 10^{-17} \text{ J}$

Sol.(A) $\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

Ex.16 Calculate the wave-number of lines having the frequency of 5×10^{16} cycles per sec -

Sol. Given $c = 3 \times 10^8 \text{ m/sec}$

$$v = 5 \times 10^{16} \text{ cycles/sec}$$

$$= ?$$

We know that

$$\bar{\nu} = \frac{v}{c} = \frac{5 \times 10^{16}}{3 \times 10^8} = 1.666 \times 10^8 \text{ m}^{-1}$$

EXERCISE (Level-1)

Question
based on

Sub-Atomic particles and Dalton's atomic theory

- Q.1** Proton is -
 (A) Nucleus of deuterium
 (B) Ionised hydrogen molecule
 (C) Ionised hydrogen atom
 (D) An α -particle
- Q.2** Which is not deflected by magnetic field -
 (A) Neutron (B) Positron
 (C) Proton (D) Electron
- Q.3** According to Dalton's atomic theory, an atom can -
 (A) Be created
 (B) Be destroyed
 (C) take part in a chemical reaction
 (D) None of these
- Q.4** Arrange α -particle(α), electron (e^-), proton(p) and neutron (n) in increasing order of their e/m value (specific charge, consider magnitude only not sign) -
 (A) $\alpha < e^- < p < n$ (B) $n < \alpha < p < e^-$
 (C) $n < p < \alpha < e^-$ (D) $e^- < p < n < \alpha$

Question
based on

Rutherford's Experiment

- Q.5** Rutherford's alpha particle scattering experiment eventually led to the conclusion that -
 (A) mass and energy are related
 (B) electrons occupy space around the nucleus
 (C) neutrons are buried deep in the nucleus
 (D) the point of impact with matter can be precisely determined
- Q.6** Which of the following conclusion could not be derived from Rutherford's α -particle scattering experiment ?
 (A) Most of the space in the atom is empty
 (B) The radius of the atom is about 10^{-10} m while that of nucleus is 10^{-15} m.

- (C) Electrons move in a circular path of fixed energy called orbits
 (D) Electrons and the nucleus are held together by electrostatic forces of attraction

Question
based on

Electromagnetic waves, hydrogen spectra & concept of quantization

- Q.7** The line spectra of two elements are not identical because -
 (A) the elements do not have the same number of neutrons
 (B) they have different mass number
 (C) their outermost electrons are at different energy levels
 (D) they have different valencies
- Q.8** A certain radio station broadcasts on a frequency of 980 kHz (kilohertz). What is the wavelength of electromagnetic radiation broadcast by the radio station ?
 (A) 306 m (B) 3.06 m
 (C) 30.6 m (D) 3060 m
- Q.9** Calculate the wavelength of the spectral line when the electron in the hydrogen atom undergoes a transition from fourth energy level to second energy level ?
 (A) 4.86 nm (B) 486 nm
 (C) 48.6 nm (D) 4860 nm
- Q.10** The wave number of the first line of Balmer series of hydrogen is 15200 cm^{-1} . The wave number of the corresponding line of Li^{2+} ion is-
 (A) 15200 cm^{-1} (B) 60800 cm^{-1}
 (C) 76000 cm^{-1} (D) 136800 cm^{-1}
- Q.11** The frequency of one of the lines in Paschen series of a hydrogen atom is $2.34 \times 10^{14} \text{ Hz}$. The higher orbit, n_2 , which produces this transitions is -
 (A) three (B) four
 (C) six (D) five

- Q.12** In hydrogen spectrum, the series of lines appearing in ultra violet region of electromagnetic spectrum are called -
 (A) Lyman lines (B) Balmer lines
 (C) Pfund lines (D) Brackett lines
- Q.13** Which of the following series of lines in the atomic spectrum of hydrogen appear in the visible region ?
 (A) Lyman (B) Paschen
 (C) Brackett (D) Balmer
- Q.14** Which of the following is not correct according to Planck's quantum theory ?
 (A) Energy is emitted or absorbed discontinuously
 (B) Energy of a quantum is directly proportional to its frequency
 (C) A photon is also a quantum of light
 (D) Energy less than a quantum can also be emitted or absorbed
- Q.15** To which electronic transition between Bohr orbits in hydrogen, the second line in the Balmer series belongs ?
 (A) $3 \rightarrow 2$ (B) $4 \rightarrow 2$
 (C) $5 \rightarrow 2$ (D) $6 \rightarrow 2$

Question
based on

Bohr's atomic model

- Q.16** The ratio of the radii of first three Bohr orbits is
 (A) 1 : 05 : 3 (B) 1 : 2 : 3
 (C) 1 : 4 : 9 (D) 1 : 8 : 27
- Q.17** The ionization energy of per mole of hydrogen atom in terms of Rydberg constant (R_H) is given by the expression -
 (A) $R_H hc$ (B) $R_H c$
 (C) $2 R_H hc$ (D) $R_H N_A hc$
- Q.18** The frequency of first line of Balmer series in hydrogen atom is ν_0 . The frequency of corresponding line emitted by singly ionised helium atom is -
 (A) $2\nu_0$ (B) $4\nu_0$
 (C) $\nu_0/2$ (D) $\nu_0/4$

- Q.19** Energy of third orbit of Bohr's atom is -
 (A) -13.6 eV (B) -3.4 eV
 (C) -1.51 eV (D) None of the three
- Q.20** If the radius of first Bohr orbit be a_0 , then the radius of the third orbit would be -
 (A) $3 \times a_0$ (B) $6 \times a_0$
 (C) $9 \times a_0$ (D) $1/9 \times a_0$
- Q.21** In H-atom electron jumps from 3rd to 2nd energy level, the energy released is -
 (A) $3.03 \times 10^{-19} \text{ J/atom}$
 (B) $1.03 \times 10^{-19} \text{ J/atom}$
 (C) $3.03 \times 10^{-12} \text{ J/atom}$
 (D) $6.06 \times 10^{-19} \text{ J/atom}$
- Q.22** The ratio of ionization energy of H and Be^{+3} is-
 (A) 1 : 1 (B) 1 : 3
 (C) 1 : 9 (D) 1 : 16
- Q.23** The ionization energy of hydrogen atom (in the ground state) is $x \text{ kJ}$. The energy required for an electron to jump from 2nd orbit to the 3rd orbit will be -
 (A) $x/6$ (B) $5x$
 (C) $7.2x$ (D) $5x/36$
- Q.24** In two H atoms X and Y the electrons move around the nucleus in circular orbits of radius r and $4r$ respectively. The ratio of the times taken by them to complete one revolution is -
 (A) 1 : 4 (B) 1 : 2
 (C) 1 : 8 (D) 2 : 1

Question
based on

Photoelectric effect, Dual Nature of electron & Heisenberg's uncertainty principle

- Q.25** If threshold wavelength (λ^0) for ejection of electron from metal is 330 nm, then work function for the photoelectric emission is -
 (A) $1.2 \times 10^{-18} \text{ J}$ (B) $1.2 \times 10^{-20} \text{ J}$
 (C) $6 \times 10^{-19} \text{ J}$ (D) $6 \times 10^{-12} \text{ J}$
- Q.26** The kinetic energy of the electron emitted when light of frequency $3.5 \times 10^{15} \text{ Hz}$ falls on a metal surface having threshold frequency $1.5 \times 10^{15} \text{ Hz}$ is ($h = 6.6 \times 10^{-34} \text{ Js}$)
 (A) $1.32 \times 10^{-18} \text{ J}$ (B) $3.3 \times 10^{-18} \text{ J}$
 (C) $6.6 \times 10^{-19} \text{ J}$ (D) $1.98 \times 10^{-19} \text{ J}$

EXERCISE (Level-2)

- Q.1** What is the maximum number of electrons in an atom that can have the quantum numbers $n = 4, m_\ell = +1$?
 (A) 4 (B) 15
 (C) 3 (D) 6
- Q.2** Arrange the orbitals of H-atom in the increasing order of their energy - $3p_x, 2s, 4d_{xy}, 3s, 4p_z, 3p_y, 4s$
 (A) $2s < 3s = 3p_x = 3p_y < 4s = 4p_z = 4d_{xy}$
 (B) $2s < 3s < 3p_x = 3p_y < 4s = 4p_z = 4d_{xy}$
 (C) $2s < 3s < 3p_x = 3p_y < 4s = 4p_z = 4d_{xy}$
 (D) $2s < 3s < 3p_x = 3p_y < 4s < 4p_z < 4d_{xy}$
- Q.3** If the I.P. of Li^{+2} is 122.4 eV. Find out 6th I.P. of carbon -
 (A) $122.4 \times 4\text{eV}$ (B) $122.4 \times 2\text{eV}$
 (C) $122.4 \times 3\text{eV}$ (D) $122.4 \times 5\text{eV}$
- Q.4** The energy difference between two electronic states is 46.12 kcal/mole. What will be the frequency of the light emitted when an electron drops from the higher to the lower energy state (Planck's constant = 9.52×10^{-14} kcal sec mole⁻¹)
 (A) 4.84×10^{15} cycles sec⁻¹
 (B) 4.84×10^{-5} cycles sec⁻¹
 (C) 4.84×10^{-12} cycles sec⁻¹
 (D) 4.84×10^{14} cycles sec⁻¹
- Q.5** If the kinetic energy of an electron is increased 4 times, the wavelength of the de Broglie wave associated with it would become :
 (A) 4 times (B) 2 times
 (C) $\frac{1}{2}$ times (D) $\frac{1}{4}$ times
- Q.6** Multiple of fine structure of spectral lines is due to-
 (A) Presence of main energy levels
 (B) Presence of sub-levels
 (C) Presence of electronic configuration
 (D) Is not a characteristics of the atom.
- Q.7** Wave mechanical mode of the atom depends upon-
 (A) de-Broglie concept of dual nature of electron
 (B) Heisenberg uncertainty principle
 (C) Schrodinger uncertainty principle
 (D) All
- Q.8** Calculate total no. of e^- having $m = 0$ in Cr atom -
 (A) 12 (B) 13
 (C) 5 (D) 24
- Q.9** Which of the following subshell can accommodate as many as 10 electrons -
 (A) 2d (B) 3d
 (C) $3d_{xy}$ (D) $3d_{z^2}$
- Q.10** How many spherical nodes are present in a 4s orbital in hydrogen atom -
 (A) 0 (B) 1
 (C) 2 (D) 3
- Q.11** Assuming the velocity to be same which subatomic particle possesses smallest de-Broglie wavelength -
 (A) An electron
 (B) A proton
 (C) An α -particle
 (D) All have same wavelength
- Q.12** I.P. of hydrogen atom is equal to 13.6 eV. What is the energy required for the process :
 $\text{He}^+ + \text{energy} \longrightarrow \text{He}^{+2} + e^-$
 (A) $2 \times 13.6 \text{ eV}$ (B) $1 \times 13.6 \text{ eV}$
 (C) $4 \times 13.6 \text{ eV}$ (D) None of these
- Q.13** If elements with principal quantum number $n > 4$ is not allowed in nature, the number of possible elements would be -
 (A) 60 (B) 32
 (C) 64 (D) 50
- Q.14** If the value of $(n + \ell)$ is not > 3 , then the maximum number of electrons in all the orbitals would be -
 (A) 12 (B) 10
 (C) 2 (D) 6
- Q.15** It is not possible to explain the Pauli's exclusion principle with the help of this atom -
 (A) B (B) Be
 (C) C (D) H
- Q.16** How fast is an electron moving if it has a wavelength equal to the distance it travels in one second -
 (A) $\sqrt{\frac{h}{m}}$ (B) $\sqrt{\frac{m}{h}}$ (C) $\sqrt{\frac{h}{p}}$ (D) $\sqrt{\frac{h}{2(\text{KE})}}$

- Q.17** An atom has a mass of 0.02 kg & uncertainty in its velocity is 9.218×10^{-6} m/s then uncertainty in position is ($h = 6.626 \times 10^{-34}$ J - s)
 (A) 2.86×10^{-28} m (B) 2.86×10^{-32} cm
 (C) 1.5×10^{-27} m (D) 3.9×10^{-10} m
- Q.18** Energy of H-atom in the ground state is -13.6 eV, Hence energy in the second excited state is –
 (A) -6.8 eV (B) -3.4 eV
 (C) -1.51 eV (D) -4.3 eV
- Q.19** Uncertainty in position of a particle of 25 g in space is 10^{-5} m. Hence uncertainty in velocity (ms^{-1}) is (Planck's constant $h = 6.6 \times 10^{-34}$ Js)
 (A) 2.1×10^{-28} (B) 2.1×10^{-34}
 (C) 0.5×10^{-34} (D) 5.0×10^{-24}
- Q.20** The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{\ell(\ell+1)} \cdot \frac{h}{2\pi}$. This momentum for an s-electron will be given by
 (A) $\frac{h}{2\pi}$ (B) $\sqrt{2} \cdot \frac{h}{2\pi}$
 (C) $+\frac{1}{2} \cdot \frac{h}{2\pi}$ (D) zero
- Q.21** Which of the following sets of quantum numbers is correct for an electron in 4f orbital?
 (A) $n = 4, \ell = 3, m = +4, s = +\frac{1}{2}$
 (B) $n = 4, \ell = 4, m = -4, s = -\frac{1}{2}$
 (C) $n = 4, \ell = 3, m = +1, s = +\frac{1}{2}$
 (D) $n = 3, \ell = 3, m = -2, s = +\frac{1}{2}$
- Q.22** Consider the ground state of Cr atom ($Z = 24$). The number of electrons with the azimuthal quantum numbers, $\ell = 1$ and 2 are, respectively
 (A) 12 and 4 (B) 12 and 5
 (C) 16 and 4 (D) 16 and 5
- Q.23** The triad of nuclei that is isotonic is –
 (A) ${}^{14}_6\text{C}, {}^{15}_7\text{N}, {}^{17}_9\text{F}$ (B) ${}^{12}_6\text{C}, {}^{14}_7\text{N}, {}^{19}_9\text{F}$
 (C) ${}^{14}_6\text{C}, {}^{14}_7\text{N}, {}^{17}_9\text{F}$ (D) ${}^{14}_6\text{C}, {}^{14}_7\text{N}, {}^{19}_9\text{F}$
- Q.24** The orbital angular momentum of an electron in 2s orbital is –
 (A) $+\frac{1}{2} \cdot \frac{h}{2\pi}$ (B) zero
 (C) $\frac{h}{2\pi}$ (D) $\sqrt{2} \cdot \frac{h}{2\pi}$
- Q.25** In potassium atom, electronic energy level is in the following order –
 (A) $4s > 3d$ (B) $4s < 2p$
 (C) $4s < 3d$ (D) $4s > 4p$
- Q.26** Which of the following has maximum number of unpaired electron?
 (A) Mg^{2+} (B) Ti^{3+}
 (C) V^{3+} (D) Fe^{2+}
- Q.27** The first use of quantum theory to explain the structure of atom was made by –
 (A) Heisenberg (B) Bohr
 (C) Planck (D) Einstein
- Q.28** For a d-electron, the orbital angular momentum is –
 (A) $\sqrt{6} \hbar$ (B) $\sqrt{2} \hbar$
 (C) \hbar (D) $2 \hbar$
- Q.29** Which of the following statement is not correct?
 (A) The electronic configuration of Cr is $[\text{Ar}] 3d^5 4s^1$. (Atomic No. of Cr = 24)
 (B) The magnetic quantum number may have a negative value
 (C) In silver atom, 23 electrons have a spin of one type of 24 of the opposite type (Atomic No. of Ag = 47)
 (D) The electronic configuration of Cr is $[\text{Ar}] 3d^4 4s^2$
- Q.30** The energy of an electron in the first Bohr orbit of H atom is -13.6 eV. The possible energy value (s) of the excited state(s) for electrons in Bohr orbits of hydrogen is (are) –
 (A) -3.4 eV (B) -4.2 eV
 (C) -6.8 eV (D) $+6.8$ eV
- Q.31** The energy of the electron in the first orbit of He^+ is -871.6×10^{-20} J. The energy of the electron in the first orbit of hydrogen would be
 (A) -871.6×10^{-20} J (B) -435.8×10^{-20} J
 (C) -217.9×10^{-20} J (D) -108.9×10^{-20} J
- Q.32** The number of nodal planes in a p_x orbitals is –
 (A) two (B) one
 (C) three (D) zero

EXERCISE (Level-3)

Part-A : Multiple correct answer type questions

Q.1 Which of the following properties is/are proportional to the energy of the electromagnetic radiation ?

- (A) Frequency (B) Wave number
(C) Wavelength (D) Number of photons

Q.2 Which of the following statements are incorrect?

- (A) There are five unpaired electrons in $(n-1)d$ suborbit in Fe^{3+}
(B) Fe^{3+} , Mn^{+} and Cr all having 24 electrons will have same value of magnetic moment
(C) Copper (I) chloride is coloured salt
(D) Every coloured ion is paramagnetic

Q.3 Which is not the correct orbital notation if the wave function is –

$$\psi = \frac{1}{81\sqrt{6\pi}} \left(\frac{1}{a_0} \right)^{3/2} \sigma^2 e^{-\sigma/3} (3\cos^2\theta - 1);$$

Here $\sigma = r/a_0$ and $a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$

- (A) 4s (B) $2p_x$
(C) $3p_y$ (D) $3d_z^2$

Q.4 Which of the following orbitals have no spherical nodes ?

- (A) 1s (B) 2s
(C) 2p (D) 3p

Q.5 In which of the following sets of orbitals, electrons have equal orbital angular momentum ?

- (A) 1s and 2s (B) 2s and 2p
(C) 2p and 3p (D) 3p and 3d

Q.6 Which of the following sets of quantum number are correct ?

- (A) $n = 3, \ell = 2, m = +1, s = +\frac{1}{2}$
(B) $n = 3, \ell = 3, m = +3, s = +\frac{1}{2}$

(C) $n = 4, \ell = 0, m = 0, s = -\frac{1}{2}$

(D) $n = 5, \ell = 2, m = +4, s = -\frac{1}{2}$

Q.7 Rutherford's experiment established that :

- (A) Inside the atom there is a heavy positive centre
(B) Nucleus contains protons and neutrons
(C) Most of the space in the atoms is empty
(D) Size of the nucleus is very small

Q.8 Which of the following statements are incorrect ?

- (A) For designating orbitals three quantum numbers are needed
(B) The second ionization energy of helium is 4 times, the first ionization of hydrogen
(C) The third ionization energy of lithium is 9 times, the first ionization of hydrogen
(D) Radius of third orbit of Li^{2+} is 3 times the radius of third orbit of hydrogen atom

Q.9 Which of the following statements (regarding an atom of H) are correct ?

- (A) Kinetic energy of the electron is maximum in the first orbit
(B) Potential energy of the electron is maximum in the first orbit
(C) Radius of the second orbit is four times the radius of the first orbit
(D) Various energy levels are equally spaced

Q.10 Which of the following transition in H-atom would result in emission of radiations of same frequency ?

- (A) $4s \rightarrow 3p$ (B) $4d \rightarrow 3p$
(C) $5s \rightarrow 4s$ (D) $3s \rightarrow 2p$

Q.11 The radial distribution functions $[P(r)]$ is used to determine the most probable radius, which is used to find the electron in a given orbital

$\frac{dP(r)}{dr}$ for 1s-orbital of hydrogen like atom having atomic number Z , is

$$\frac{dP}{dr} = \frac{4Z^3}{a_0^3} \left(2r - \frac{2Zr^2}{a_0} \right) e^{-2Zr/a_0}$$

Then which of the following statements is/are correct ?

- (A) At the point of maximum value of radial distribution function $\frac{dP(r)}{dr} = 0$; One antinode is present
- (B) Most probable radius of Li^{2+} is $\frac{a_0}{3}$ pm
- (C) Most probable radius of He^+ is $\frac{a_0}{2}$ pm
- (D) Most probable radius of hydrogen atom is a_0 pm

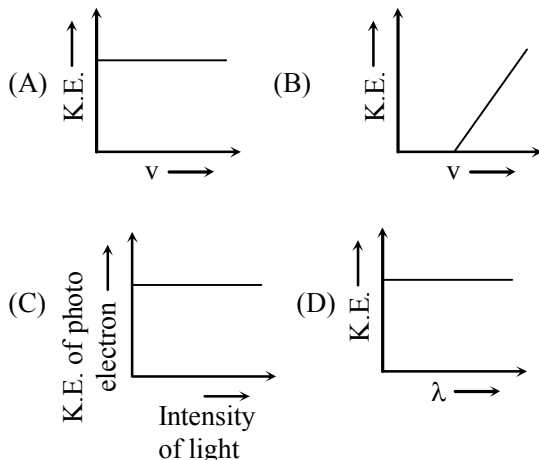
Q.12 Select the correct statement (s) -

- (A) An orbital with $\ell = 0$ is symmetrical about the nucleus
- (B) An orbital with $\ell = 0$ is spherically symmetrical about the nucleus
- (C) $3d_{z^2}$ is spherically symmetrical about the z-axis
- (D) All are correct

Q.13 Select the correct statement (s) -

- (A) Radial function depends only on the direction, and is independent on quantum number n only the nucleus
- (B) Angular function depends only on the direction and is independent to the distance from the nucleus
- (C) $\Psi^2(r, \theta, \phi)$ is the probability density of finding the electron at a particular point in space
- (D) Radial distribution function ($4\pi r^2 R^2$) gives the probability of the electron being present at a distance r from the nucleus

Q.14 Which is correct graph ?



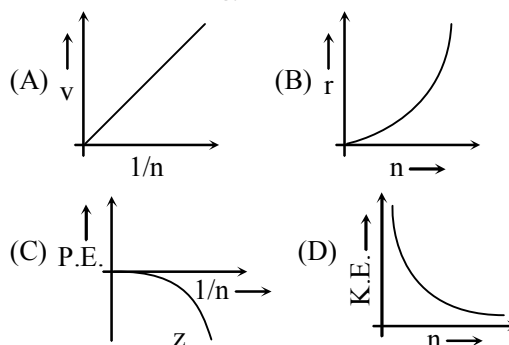
Q.15 Select the correct curve (s) -

If v = Velocity of electron in Bohr's orbit

r = Radius of electron in Bohr's orbit

P.E. = Potential energy of electron in Bohr's orbit

K.E. = Kinetic energy of electron in Bohr's orbit



Q.16 Select the correct statement(s) -

- (A) An electron near the nucleus is attracted by the nucleus and has a low potential energy
- (B) According to Bohr's theory, an electron continuously radiate energy if it stayed in one orbit
- (C) Bohr's model could not explain the spectra of multielectron atoms
- (D) Bohr's model was the first atomic model based on quantisation of energy

Q.17 Choose the correct statement(s) -

- (A) The shape of an atomic orbital depends upon azimuthal quantum number
- (B) The orientation of an atomic orbital depends upon the magnetic quantum number
- (C) The energy of an electron in an atomic orbital of multi-electron atom depends upon principal quantum number only
- (D) The number of degenerate atomic orbital of one type depends upon the value of azimuthal and magnetic quantum number

Q.18 For radial probability curves, which of the following is/are correct ?

- (A) The number of maxima in 2s orbital are two
- (B) The number of spherical or radial nodes is equal to $n-l-1$
- (C) The number of angular nodes are ' l '
- (D) $3d_z^2$ has 3 angular nodes

- Q.19** Select the correct statement(s) -
- (A) Radial distribution function indicates that there is a higher probability of finding the 3s electron close to the nucleus than in case of 3p and 3d electrons
 - (B) Energy of 3s orbital is less than for the 3p and 3d orbitals
 - (C) At the node, the value of the radial function changes from positive to negative
 - (D) The radial function depends upon the quantum numbers n and l
- Q.20** Choose the incorrect statement(s) -
- (A) For a particular orbital in hydrogen atom, the wave function may have negative value
 - (B) Radial probability distribution function though may have zero value but can never have negative value
 - (C) $3d_{x^2-y^2}$ orbital has two angular nodes and one radial node
 - (D) yz and xz planes are nodal planes for d_{xy} orbital
- Q.21** Select the correct statement(s) -
- (A) Heisenberg's principle is applicable to stationary e^-
 - (B) Pauli's exclusion principle is not applicable to photons
 - (C) For an e^- , the product of velocity and principal quantum number will be independent of principal quantum number
 - (D) Quantum numbers l and m determine the value of angular wave function
- Q.22** Choose the correct statements among the following -
- (A) A node is a point in space where the wave-function Ψ has zero amplitude
 - (B) The number of maxima (peaks) in radial distribution in $n-l$
 - (C) Radial probability is $4\pi r^2 R_{n,\ell}^2(r)$
 - (D) Ψ^2 represents probability of finding $3p_y$ orbital
- Q.23** Select the correct statement(s) regarding $3p_y$ orbital
- (A) Total no. of nodes are 2
 - (B) Number of maxima in the curve $4\pi r^2 R^2(r)$ Vs r is one

- (C) Quantum no. n , l and m for orbital may be, 3, 1, -1 respectively
- (D) The magnetic quantum number may have a positive value

Part-B : Assertion Reason type Questions

The following questions 24 to 27 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

- (A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
 - (B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.
 - (C) If Assertion is true but the Reason is false.
 - (D) If Assertion is false but Reason is true
- Q.24** **Assertion** : The charge to mass ratio of the particles in anode rays depends on nature of the gas taken in the discharge tube.
Reason : The particles in anode rays carry positive charge.
- Q.25** **Assertion** : s-orbital cannot accommodate more than two electrons.
Reason : s-orbitals are spherically symmetrical.
- Q.26** **Assertion** : Kinetic energy of photoelectrons is directly proportional to the intensity of the incident radiation
Reason : Each photon of light causes the emission of only one photo electron.
- Q.27** **Assertion** : The existence of three unpaired electrons in phosphorous atom can be explained on the basis of Hund's rule.
Reason : According to Hund's rule, the degenerate orbitals are first singly occupied and only then pairing takes place.

Part-C : Column Matching type Questions

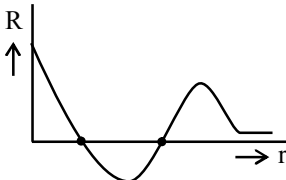
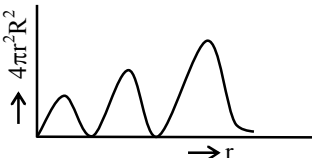
Q.28

Column-A	Column-B
(A) If P.E. = -13.6 eV	(i) 21
(B) Ionization energy of electron from 2 nd shell of Na^{10+}	(ii) 10
(C) Number of spectral lines when electron 7 th to 3 rd shell	(iii) Total energy = -6.8 eV
(D) Number of spectral lines when electron comes from 7 th shell to 1 st shell	(iv) 411.4 eV

Q.29

Column-A	Column-B
(A) Orbitals having equal energy	(i) 3p, 3d
(B) Orbitals having zero orbital angular momentum	(ii) 2s and 3s
(C) Orbitals with only one spherical node	(iii) Degenerate orbitals
(D) Orbitals having directional character	(iv) 2s and 3p

Q.30 Match the column :

Column-A	Column-B
(A) 	(i) 3p _z
(B) 	(ii) 4p _y
(C) Angular probability is dependent on θ and ϕ	(iii) 3s
(D) At least one angular node is present	(iv) 3p _x

Part-D : Passage based objective questions

Passage-1 (Ques. 31 to 33)

Electron moves around the nucleus in circular orbitals in fixed energy paths. As far as electron moves in these orbits neither energy is absorbed nor liberated. But when electron move from lower energy level to higher energy level energy is absorbed while when it comes back from higher energy level to lower energy level energy is liberated in the form of photon & a spectral line is formed. Corresponding to different possible transitions different lines are formed which form the particular series viz. lyman, balmer, paschen, bracket, pfund, homphery etc. Suppose e^- in hydrogen atom is present in 10th excited state, then answer the following questions based on paragraph :

Q.31 If electron present in 10th excited state liberate one visible quanta then next quanta liberated will correspond to following transition -

- (A) $10 \rightarrow 2$ (B) $11 \rightarrow 2$
(C) $11 \rightarrow 1$ (D) $2 \rightarrow 1$

Q.32 Total number of spectral lines which can be obtained during the transition to ground level -

- (A) 45 (B) 55
(C) 66 (D) 36

Q.33 Minimum value of wavelength that can be obtained during the transition -

- (A) $\frac{121}{120R}$ (B) $\frac{11}{10R}$
(C) $\frac{100}{99R}$ (D) $\frac{10}{9R}$

Where R is Rydberg constant.

Passage-2 (Ques. 34 to 36)

Suppose Bohr theory is applicable to a negatively charged particle of mass $2m_e$ and charge $2e$ removing around the nucleus of positive charge Ze . Let r_1 , v_1 and E_1 be the radius of the orbit speed of the particle in the orbit and energy of the particle in the orbit respectively. The values for the electron revolving in the corresponding orbit are r , v and E respectively.

Q.34 Which of the following expression regarding the ratio of radii is correct ?

- (A) $\frac{r_1}{r} = 2$ (B) $r_1/r = \frac{1}{2}$
 (C) $\frac{r_1}{r} = 4$ (D) $\frac{r_1}{r} = \frac{1}{4}$

Q.35 Which of the following expression regarding the ratio of speeds is correct?

- (A) $\frac{v_1}{v} = 2$ (B) $\frac{v_1}{v} = \frac{1}{2}$
 (C) $\frac{v_1}{v} = 4$ (D) $\frac{v_1}{v} = \frac{1}{4}$

Q.36 Which of the following expression regarding the ratio of energies is correct ?

- (A) $\frac{E_1}{E} = 4$ (B) $\frac{E_1}{E} = \frac{1}{4}$
 (C) $\frac{E_1}{E} = 8$ (D) $\frac{E_1}{E} = \frac{1}{8}$

Passage-3 (Ques. 37 to 39)

Assume that there were four possible values $(-1, \frac{1}{2}, +\frac{1}{2}, +1)$ for the spin quantum number m_s . Principal quantum number n is defined as usual. However, quantum number ℓ & m_ℓ are defined as follows :

ℓ : 1 to $(n+1)$ in integral steps

m_ℓ : $-\ell/2$ to $+\ell/2$ (including zero, if any) in integral steps.

The orbitals corresponding to $\ell = 1, 2, 3 \dots$ designated as A, B, C.....respectively. Further, the values of m_ℓ for a given value of ℓ give the number of sub-orbitals in an orbital.

The principles for filling electrons in the shells remain unchanged. The order of energies of various orbitals is : $A < B < C \dots$ for the same shell.

Q.37 The second period would begin with -

- (A) Fluorine (B) Sodium
 (C) Calcium (D) Scandium

Q.38 If Aufbau's principle is not to be violated i.e. $(n + \ell)$ rule must be followed, the outermost electronic configuration of an element with at. no. 100 would be -

- (A) $3B^8 4A^4$ (B) $3C^{16} 4A^8$
 (C) $3C^{12} 4B^8$ (D) $4B^{12} 5A^8$

Q.39 The number of sub-orbitals & the maximum number of electrons that can be filled in an E-orbitals are respectively -

- (A) 6, 24
 (B) 5, 20
 (C) 7, 28
 (D) can't be determined

Passage-4 (Q.40 to Q.42)

Orbital wave function Ψ can be given as

$$\Psi(r, \theta, \phi) = R(r) \cdot \theta(\theta) \cdot \phi(\phi)$$

For various orbitals of H-atom and H-like atoms values of R (radial wave function) are

For 1s-orbital : $R_{1s} = 2 \left(\frac{z}{a_0} \right)^{3/2} e^{-zr/a_0}$

For 2s-orbital :

$$R_{2s} = \left(\frac{z}{2a_0} \right)^{3/2} \left(2 - \frac{zr}{a_0} \right) e^{-zr/2a_0}$$

For 1p-orbital

$$R_{2p} = \frac{1}{\sqrt{3}} \left(\frac{z}{2a_0} \right)^{3/2} \left(\frac{zr}{a_0} \right) \times e^{-zr/2a_0}$$

For 3s-orbital

$$R_{3s} = \frac{2}{3} \left(\frac{z}{3a_0} \right)^{3/2} \left(3 - \frac{2zr}{a_0} + \frac{2z^2 r^2}{9a_0^2} \right) \times e^{-zr/3a_0}$$

etc. Here $a_0 = \text{bohr radius} = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2}$

$z = \text{atomic number}$

Similarly angular functions θ and ϕ can also be given

For s-orbital θ and ϕ can be given as

$$\theta(\theta) = \frac{1}{\sqrt{2}} \text{ and } \phi(\phi) = \frac{1}{\sqrt{2\pi}}$$

So for 1s-orbital Ψ can be given as -

$$\begin{aligned} \Psi_{1s} &= R_{1s} \cdot \theta_{1s} \cdot \phi_{1s} \\ &= \frac{1}{\sqrt{\pi}} \left(\frac{z}{a_0} \right)^{3/2} e^{-zr/a_0} \end{aligned}$$

Q.40 Probability density of finding an electron at distance r from nucleus in H-atom (in ground state) is -

- (A) Ψ (B) R^2
 (C) $\frac{1}{\pi} \left(\frac{Z}{a_0} \right)^3 e^{-2Zr/a_0}$ (D) None of these

Q.41 Value of r at which radial node is found for 2s-orbital is -

- (A) a_0 (B) $2a_0$
 (C) $3a_0$ (D) $4a_0$

Q.42 Radial nodes for 3s-orbital is/are at -

- (A) $r = 1.9 a_0$ (B) $r = 7.1 a_0$
 (C) $r = 2a_0$ (D) Both (A) and (B)

Part-E : Numeric Response Type Questions

Q.43 The line at 434 nm in the Balmer series of the hydrogen spectrum corresponds to a transition of an electron from the n^{th} to second Bohr orbit. What is the value of n ?

Q.44 A particle of charge equal to that of an electron and mass 400 times the mass of the electron moves in a circular orbit around the nucleus of charge $+4e$. Assuming that the Bohr model of the atom is applicable to this system find the value of n for which the

Part-F : Subjective Type Questions

Q.45 When would the wavelength associated with an electron be equal to wavelength of proton?
 (mass of $e = 9 \times 10^{-28}$ g;
 mass of proton = 1.6725×10^{-24} g)

Q.46 Point out the angular momentum of an electron in
 (a) 4s orbital
 (b) 3p orbital
 (c) 4th orbit

Q.47 (a) The wave number of the first line in the Balmer series of Be^{3+} is $2.43 \times 10^5 \text{ cm}^{-1}$. What is the wave number of the second line of the Paschen series of Li^{2+} ?
 (b) In ions like He^+ , Li^{2+} , Be^{3+} how and why does the value of the Rydberg constant vary?

Q.48 Calculate the wavelength in \AA of the photon that is emitted when an electron in Bohr orbit with $n = 2$ returns to orbit with $n = 1$ in H atom. The ionisation potential of the ground state of H-atom is 2.17×10^{-11} erg.

Q.49 Two particles A and B are in motion. If the wavelength associated with the particle A is 5×10^{-8} m, calculate the wavelength of particle B if its momentum is half of A.

Q.50 The first ionization energy of H is 21.79×10^{-19} J. Determine the second ionization energy of He atom.

Q.51 How many times larger is the spacing between the energy levels with $n = 3$ and $n = 8$ spacing between the energy level with $n = 8$ and $n = 9$ for the hydrogen atom?

Q.52 Calculate the minimum uncertainty in velocity of a particle of mass 1.1×10^{-27} kg if uncertainty in its position is 3×10^{-10} cm.

Q.53 Calculate the number of photons emitted in 10 hours by a 60 W sodium lamp.
 ($\lambda_{\text{photon}} = 5893 \text{ \AA}$).

Q.54 Calculate total spin, magnetic moment for the atoms having atomic number 7, 24, 34 and 36.

Q.55 Magnetic moment of X^{3+} ion of 3d series is $\sqrt{35}$ B.M. What is atomic number of X^{3+} ?

Q.56 When a certain metal was irradiated with light of frequency 3.2×10^{16} Hz, the photoelectrons emitted had twice the kinetic energy as did photoelectrons emitted when the same metal was irradiated with light of frequency 2.0×10^{16} Hz. Calculate ν_0 for the metal.

Q.57 Calculate the circumference of the 4th Bohr orbit for an electron travelling with a velocity of $2.19 \times 10^6 \text{ m/s}$.

Q.58 1.53 g of hydrogen is excited by irradiation. At a certain instant, 10% of the atoms are at the excited level of energy -328 kJ mol^{-1} and 2% of the atoms are at the excited level of energy $-146 \text{ kJ mole}^{-1}$. The remaining atoms are in the ground state. Calculate how much energy will be evolved when all the excited atoms return to the ground state.

EXERCISE (Level-4)

Old Examination Questions

Section-A [JEE Main]

- Q.1** In a multi-electron atom, which of the following orbitals described by the three quantum numbers will have the same energy in the absence of magnetic and electric fields ? [AIEEE- 2005]
 (a) $n = 1, \ell = 0, m = 0$ (b) $n = 2, \ell = 0, m = 0$
 (c) $n = 2, \ell = 1, m = 1$ (d) $n = 3, \ell = 2, m = 1$
 (e) $n = 3, \ell = 2, m = 0$
 (A) (b) and (c) (B) (a) and (b)
 (C) (d) and (e) (D) (c) and (d)
- Q.2** Of the following sets which one does NOT contain isoelectronic species ? [AIEEE- 2005]
 (A) CN^- , N_2 , C_2^{2-}
 (B) PO_4^{3-} , SO_4^{2-} , ClO_4^-
 (C) BO_3^{3-} , CO_3^{2-} , NO_3^-
 (D) SO_3^{2-} , CO_3^{2-} , NO_3^-
- Q.3** According to Bohr's theory, the angular momentum of an electron in 5th orbit is - [AIEEE 2006]
 (A) $1.0 \hbar/\pi$ (B) $10 \hbar/\pi$
 (C) $2.5 \hbar/\pi$ (D) $25 \hbar/\pi$
- Q.4** Uncertainty in the position of an electron (mass = 9.1×10^{-31} kg) moving with a velocity 300 m/s, accurate upto 0.001 %, will be ($\hbar = 6.63 \times 10^{-34}$ Js) [AIEEE 2006]
 (A) 5.76×10^{-2} m (B) 1.92×10^{-2} m
 (C) 3.84×10^{-2} m (D) 19.2×10^{-2} m
- Q.5** Which of the following sets of quantum numbers represents the highest energy of an atom ? [AIEEE 2007]
 (A) $n = 3, \ell = 1, m = 1, s = +\frac{1}{2}$
 (B) $n = 3, \ell = 2, m = 1, s = +\frac{1}{2}$
 (C) $n = 4, \ell = 0, m = 0, s = +\frac{1}{2}$
 (D) $n = 3, \ell = 0, m = 0, s = +\frac{1}{2}$
- Q.6** The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol⁻¹. The energy required to excite the electron in the atom from $n = 1$ to $n = 2$ is [AIEEE 2008]
 (A) 6.56×10^5 J mol⁻¹
 (B) 7.56×10^5 J mol⁻¹
 (C) 9.84×10^5 J mol⁻¹
 (D) 8.51×10^5 J mol⁻¹
- Q.7** In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is ($\hbar = 6.6 \times 10^{-34}$ kg m²s⁻¹, mass of electron, $m_e = 9.1 \times 10^{-31}$ kg) [AIEEE 2009]
 (A) 1.52×10^{-4} m (B) 5.10×10^{-3} m
 (C) 1.92×10^{-3} m (D) 3.84×10^{-3} m
- Q.8** Calculate the wavelength (in nanometer) associated with a proton moving at 1.0×10^3 m s⁻¹ (Mass of proton = 1.67×10^{-27} kg and $\hbar = 6.63 \times 10^{-34}$ Js) - [AIEEE 2009]
 (A) 0.032 nm (B) 0.40 nm
 (C) 2.5 nm (D) 14.0 nm
- Q.9** A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at: [AIEEE 2011]
 (A) 1035 nm (B) 325 nm
 (C) 743 nm (D) 518 nm
- Q.10** The frequency of light emitted for the transition $n = 4$ to $n = 2$ of He^+ is equal to the transition in H atom corresponding to which of the following ? [AIEEE 2011]
 (A) $n = 2$ to $n = 1$ (B) $n = 3$ to $n = 2$
 (C) $n = 4$ to $n = 3$ (D) $n = 3$ to $n = 1$
- Q.11** The electrons identified by quantum numbers n and ℓ [AIEEE-2012]
 (a) $n = 4, \ell = 1$ (b) $n = 4, \ell = 0$
 (c) $n = 3, \ell = 2$ (d) $n = 3, \ell = 1$
 can be placed in order of increasing energy as -
 (A) (d) < (b) < (c) < (a) (B) (b) < (d) < (a) < (c)
 (C) (a) < (c) < (b) < (d) (D) (c) < (d) < (b) < (a)
- Q.12** The following sets of quantum numbers represents four electrons in an atom :
 (i) $n = 4, \ell = 1$ (ii) $n = 4, \ell = 0$
 (iii) $n = 3, \ell = 2$ (vi) $n = 3, \ell = 1$
 The sequence representing increasing order of energy, is : [AIEEE Online-2012]
 (A) (i) < (iii) < (ii) < (iv) (B) (ii) < (iv) < (i) < (iii)
 (C) (iv) < (ii) < (iii) < (i) (D) (iii) < (i) < (iv) < (ii)

Q.13 The limiting line in Balmer series will have a frequency of :

(Rydberg constant, $R_{\infty} = 3.29 \times 10^{15}$ cycles/s)

[AIEEE Online-2012]

- (A) $3.65 \times 10^{14} \text{ s}^{-1}$ (B) $8.22 \times 10^{14} \text{ s}^{-1}$
(C) $3.29 \times 10^{15} \text{ s}^{-1}$ (D) $5.26 \times 10^{13} \text{ s}^{-1}$

Q.14 If the kinetic energy of an electron is increased four times, the wavelength of the de-Broglie wave associated with it would become :

[AIEEE Online-2012]

- (A) Two times (B) Half
(C) One fourth (D) Four times

Q.15 Which pair of elements with the given atomic numbers is expected to have similar properties ?

[AIEEE Online-2012]

- (A) 11, 12 (B) 40, 72
(C) 20, 36 (D) 10, 28

Q.16 If the radius of first orbit of H atom is a_0 , the de-Broglie wavelength of an electron in the third orbit is :

[AIEEE Online-2012]

- (A) $6 \pi a_0$ (B) $8 \pi a_0$
(C) $2 \pi a_0$ (D) $4 \pi a_0$

Q.17 In an atom how many orbital (s) will have the quantum numbers, $n = 3$, $\ell = 2$ and $m_{\ell} = +2$?

[JEE Main Online-2013]

- (A) 5 (B) 3
(C) 1 (D) 7

Q.18 The wave number of the first emission line in the Balmer series of H-Spectrum is –
(R = Rydberg constant)

[JEE Main Online-2013]

- (A) $\frac{5}{36}R$ (B) $\frac{9}{400}R$
(C) $\frac{7}{6}R$ (D) $\frac{3}{4}R$

Q.19 The numbers of protons, electrons and neutrons in a molecule of heavy water are respectively :

[JEE Main Online-2013]

- (A) 8, 10, 11 (B) 10, 10, 10
(C) 10, 11, 10 (D) 11, 10, 10

Q.20 The de Broglie wavelength of a car of mass 1000kg and velocity 36 km/hr is :

($h = 6.63 \times 10^{-34} \text{ Js}$) [JEE Main Online-2013]

- (A) $6.626 \times 10^{-34} \text{ m}$ (B) $6.626 \times 10^{-38} \text{ m}$
(C) $6.626 \times 10^{-31} \text{ m}$ (D) $6.626 \times 10^{-30} \text{ m}$

Q.21 Energy of an electron is given by

$$E = -2.178 \times 10^{-18} \text{ J} \left(\frac{Z^2}{n^2} \right)$$

Wavelength of light required to excite an electron in an hydrogen atom from level $n = 1$ to $n = 2$ will be : ($h = 6.62 \times 10^{-34} \text{ Js}$ and $c = 3.0 \times 10^8 \text{ ms}^{-1}$)

[JEE-Main 2013]

- (A) $6.500 \times 10^{-7} \text{ m}$ (B) $8.500 \times 10^{-7} \text{ m}$
(C) $1.214 \times 10^{-7} \text{ m}$ (D) $2.816 \times 10^{-7} \text{ m}$

Q.22 The correct set of four quantum numbers for the valence electrons of rubidium atom ($Z = 37$) is -

[JEE Main 2014]

- (A) 5, 1, 0 + $\frac{1}{2}$ (B) 5, 1, 1 + $\frac{1}{2}$
(C) 5, 0, 1 + $\frac{1}{2}$ (D) 5, 0, 0 + $\frac{1}{2}$

Q.23 Given

(a) $n = 5$, $m_{\ell} = +1$

(b) $n = 2$, $\ell = 1$, $m_{\ell} = -1$, $m_s = -\frac{1}{2}$

The maximum number of electron (s) in an atom that can have the quantum numbers as given in (a) and (b) are respectively –

[JEE Main Online - 2014]

- (A) 25 and 1 (B) 8 and 1
(C) 2 and 4 (D) 4 and 1

Q.24 Ionization energy of gaseous Na atoms is 495.5 kJmol^{-1} . The lowest possible frequency of light that ionizes a sodium atom is

[JEE Main Online - 2014]

($h = 6.626 \times 10^{-34} \text{ Js}$, $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$)

- (A) $7.50 \times 10^4 \text{ s}^{-1}$ (B) $4.76 \times 10^{14} \text{ s}^{-1}$
(C) $3.15 \times 10^{15} \text{ s}^{-1}$ (D) $1.24 \times 10^{15} \text{ s}^{-1}$

Q.25 If m and e are the mass and charge of the revolving electron in the orbit of radius r for hydrogen atom, the total energy of the revolving electron will be:

[JEE Main Online - 2014]

- (A) $\frac{1}{2} \frac{e^2}{r}$ (B) $-\frac{e^2}{r}$
(C) $\frac{me^2}{r}$ (D) $-\frac{1}{2} \frac{e^2}{r}$

Q.26 The de-Broglie wavelength of a particle of mass 6.63 g moving with a velocity of 100 ms^{-1} is :

[JEE Main Online - 2014]

- (A) 10^{-33} m (B) 10^{-35} m
(C) 10^{-31} m (D) 10^{-25} m

Section-B [JEE Advanced]

- Q.1** The number of radial nodal surface in 3s and 2p [IIT-2005]
(A) 2, 0 (B) 2, 1 (C) 1, 0 (D) 0, 2
- Q.2** Give answer : [IIT-2005]
(a) For first orbit of hydrogen atom, calculate the velocity of electron ($r = a_0 = 0.529 \text{ \AA}$)
(b) Calculate the de-broglie wavelength of electron in first Bohr orbit
(c) Calculate the orbital angular momentum of 2p orbital in terms of $h/2\pi$ units
- Q.3** According to Bohr's theory, [IIT-2006]
 E_n = Total energy ;
 K_n = Kinetic energy
 V_n = Potential energy
 r_n = Radius of n^{th} orbit
Match the following :

Column-I		Column-II	
(A)	$V_n/K_n = ?$	(P)	0
(B)	If radius of n^{th} orbital is r_n , $r_n \propto E_n^x$, $x = ?$	(Q)	-1
(C)	Angular momentum in lowest orbital	(R)	-2
(D)	$\frac{1}{r^n} \propto Z^y$, $y = ?$	(S)	1

- Q.4** Match the entries in column-I with the correctly related quantum no. (s) in column-II [IIT-2008]

Column-I		Column-II	
(A)	orbital angular momentum of the electron in a hydrogen like atomic orbital	(P)	Principal quantum number
(B)	A hydrogen like one electron wave function obeying Pauli's number principle	(Q)	Azimuthal quantum number
(C)	Shape, size and orientation of hydrogen like atomic orbital	(R)	magnetic quantum number
(D)	Probability density of electron at the nucleus in hydrogen like atom	(S)	Electron spin quantum number

Passage based objective questions

Passage :1 (Ques. 5 to 7)

The hydrogen like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2 . The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

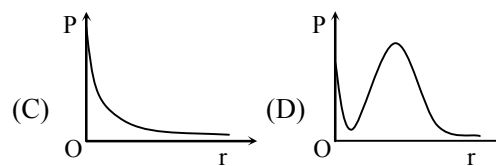
[IIT-2010]

- Q.5** The state S_1 is -
(A) 1s (B) 2s
(C) 2p (D) 3s
- Q.6** Energy of the state S_1 in units of the hydrogen atom ground state energy is -
(A) 0.75 (B) 1.50
(C) 2.25 (D) 4.50
- Q.7** The orbital angular momentum quantum number of the state S_2 is -
(A) 0 (B) 1
(C) 2 (D) 3
- Q.8** The maximum number of electrons that can have principal quantum number, $n = 3$ and spin quantum number, $m_s = -1/2$, is. [IIT-2011]
- Q.9** The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is. [IIT-2011]

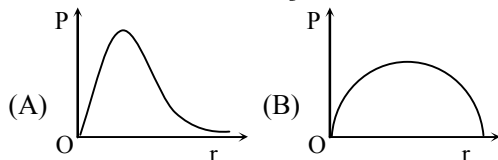
Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

- Q.10** The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius] [IIT-2012]
(A) $\frac{h^2}{4\pi^2 m a_0^2}$ (B) $\frac{h^2}{16\pi^2 m a_0^2}$
(C) $\frac{h^2}{32\pi^2 m a_0^2}$ (D) $\frac{h^2}{64\pi^2 m a_0^2}$
- Q.11** In an atom, the total number of electrons having quantum numbers $n = 4$, $|m_l| = 1$ and $m_s = -1/2$ is [JEE-Advance-2014]
- Q.12** Not considering the electronic spin, the degeneracy of the second excited state ($n = 3$) of H atom is 9, while the degeneracy of the second excited state of H^- is - [JEE-Advance-2015]

- Q.13** P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is -



[JEE-Advance-2016]



Passage :2 (Ques. 14 to 16)

Answer Q.14, Q.15 and Q.16 by appropriately matching the information given in the three columns of the following table [JEE-Advance-2017]

The wave function, Ψ_{n,ℓ,m_1} is a mathematical function whose value depends upon spherical polar coordinates (r, θ , ϕ) of the electron and characterized by the quantum numbers n, ℓ and m_1 . Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the Table, Z is atomic number and a_0 is Bohr radius		
Column 1	Column 2	Column 3
(I) 1s orbital	(i) $\Psi_{n,\ell,m_1} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P)
(II) 2s orbital	(ii) One radial node	(Q) Probability density at nucleus $\propto \frac{1}{a_0^3}$
(III) 2p _z orbital	(iii) $\Psi_{n,\ell,m_1} \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} r e^{-\left(\frac{Zr}{2a_0}\right)} \cos \theta$	(R) Probability density is maximum at nucleus
(IV) 3d _{xy} orbital	(iv) xy- plane is a nodal plane	(S) Energy needed to excite electron from n = 2 state to n = 4 state is $\frac{27}{32}$ times the energy needed to excite electron from n = 2 state to n = 6 state

- Q.14** For the given orbital in Column 1, the only **CORRECT** combination for any hydrogen-like species is
 (A) (III) (iii) (P) (B) (II) (ii) (P) (C) (IV) (iv) (R) (D) (I) (ii) (S)

- Q.15** For He^+ ion, the only **INCORRECT** combination is
 (A) (I) (i) (R) (B) (I) (i) (S) (C) (I) (iii) (R) (D) (II) (ii) (Q)

- Q.16** For hydrogen atom, the only **CORRECT** combination is
 (A) (I) (i) (S) (B) (I) (iv) (R) (C) (I) (i) (P) (D) (II) (i) (Q)

EXERCISE (Level-5)

Review Exercise

Q.1 An electron is accelerated from a very low velocity (\sim zero speed) by the application of a potential difference of V volts. If the de Broglie wavelength should change (i.e., decrease) by 1.0% what percent increase in V causes it-

Q.2 An electron first accelerated through 100 volts suffers successively two retardations (i) through 19 volts and then (ii) through 32 volts. Its de Broglie wavelengths in the three situations are respectively λ_1 , λ_2 and λ_3 . Calculate $\frac{\lambda_3 - \lambda_2}{\lambda_1}$.

Q.3 Photons having energy equivalent to binding energy of 2^{nd} state of Li^+ ion are used at metal surface of work function 10.6 eV. If the ejected electrons are further accelerated through the potential difference of 5 V then the minimum value of de-Broglie wavelength associated with the electron is -

Q.4 A hydrogen atom in its ground state absorbs a photon and goes into the first excited state. It then absorbs a second photon which just ionizes it. What is the ratio of the wavelengths of the first photon and the second photon?

Q.5 A hydrogen like atom with atomic number ' Z ' is in higher excited state of quantum number ' n '. This excited state atom can make a transition to the first excited state by successively emitting two photons of energies 10 eV and 17 eV respectively. Alternatively, the atom from the same excited state can make a transition to the 2nd excited state by emitting two photons of energies 4.25 eV and 5.95 eV respectively. The ' n ' and ' Z ' are-

Q.6 The Schrodinger wave equation for hydrogen atom is

$$\Psi(\text{radial}) = \frac{1}{16\sqrt{4}} \left(\frac{Z}{a_0} \right)^{3/2} [(\sigma - 1)(\sigma^2 - 8\sigma + 12)] e^{-\sigma/2}$$

where a_0 and Z are the constant in which answer can be expressed and $\sigma = \frac{2Zr}{a_0}$ minimum and maximum position of radial nodes from nucleus are.....respectively.

Q.7 For a hypothetical hydrogen like atom, the potential energy of the system is given by $U(r) = \frac{-Ke^2}{r^3}$, where r is the distance between the two particles. If Bohr's model of quantization of angular momentum is applicable then velocity of particle is given by:

Q.8 An element undergoes a reaction as shown: $X + 2e^- \rightarrow X^{2-}$, energy released = 30.87 eV/atom. If the energy released, is used to dissociate 4 gms of H_2 molecules, equally into H^+ and H^* , where H^* is excited state of H atoms where the electron travels in orbit whose circumference equal to four times its de Broglie's wavelength. Determine the least moles of X that would be required: Given: I.E. of H = 13.6 eV/atom, bond energy of H_2 = 4.526 eV/molecule

Q.9 In a measurement of quantum efficiency of photosynthesis in green plants, it was found that 10 quanta of red light of wavelength 6850 Å were needed to release one molecule of O_2 . The average energy storage in this process is 112 kcal/mol O_2 evolved. What is the energy conversion efficiency in this experiment? Given: 1 cal = 4.18 J; $N_A = 6 \times 10^{23}$; $h = 6.63 \times 10^{-34}$ J. s

Q.10 Find the value of wave number ($\bar{\nu}$) in terms of Rydberg's constant, when transition of electron takes place between two levels of He^+ ion whose sum is 4 and difference is 2.

ANSWER KEY

EXERCISE (Level-1)

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (C) | 2. (A) | 3. (C) | 4. (B) | 5. (B) | 6. (C) | 7. (C) |
| 8. (A) | 9. (B) | 10. (D) | 11. (D) | 12. (A) | 13. (D) | 14. (D) |
| 15. (B) | 16. (C) | 17. (D) | 18. (B) | 19. (C) | 20. (C) | 21. (A) |
| 22. (D) | 23. (D) | 24. (C) | 25. (C) | 26. (A) | | |

EXERCISE (Level-2)

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (D) | 2. (A) | 3. (A) | 4. (D) | 5. (C) | 6. (B) | 7. (D) |
| 8. (A) | 9. (B) | 10. (D) | 11. (C) | 12. (C) | 13. (A) | 14. (A) |
| 15. (D) | 16. (A) | 17. (A) | 18. (C) | 19. (A) | 20. (D) | 21. (C) |
| 22. (B) | 23. (A) | 24. (B) | 25. (C) | 26. (D) | 27. (B) | 28. (A) |
| 29. (D) | 30. (A) | 31. (C) | 32. (B) | | | |

EXERCISE (Level-3)

Part-A

- | | | | | | | |
|--------------|---------------|-------------|---------------|-------------|-------------|-------------|
| 1. (A,B,D) | 2. (B,C,D) | 3. (A,B,C) | 4. (A,C) | 5. (A,C) | 6. (A,C) | 7. (A,C,D) |
| 8. (D) | 9. (A,C) | 10. (A,B) | 11. (A,B,C,D) | 12. (A, B) | 13. (B,C,D) | 14. (B,C) |
| 15. (A,B) | 16. (A,C,D) | 17. (A,B,D) | 18. (A,B,C) | 19. (A,C,D) | 20. (B,C) | 21. (B,C,D) |
| 22. (AB,C,D) | 23. (A,B,C,D) | | | | | |

Part-B

- | | | | |
|---------|---------|---------|---------|
| 24. (B) | 25. (B) | 26. (D) | 27. (A) |
|---------|---------|---------|---------|

Part-C

- | | | | |
|----------------|------------------|-----------------|---------------------|
| 28. A → (iii); | B → (iv); | C → (ii); | D → (i) |
| 29. A → (iii); | B → (ii); | C → (iv); | D → (i) |
| 30. A → (iii); | B → (ii), (iii); | C → (ii), (iv); | D → (i), (ii), (iv) |

Part-D

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 31. (D) | 32. (B) | 33. (A) | 34. (D) | 35. (A) | 36. (C) | 37. (D) |
| 38. (B) | 39. (A) | 40. (C) | 41. (B) | 42. (D) | | |

Part-E

43. $n = 5$ 44. $n = 40$

Part-F

- | | | |
|---|--|---------------------------------------|
| 45. $\frac{v_e}{v_p} = 1.858 \times 10^3$ | 46. (a) 0 (b) $\frac{h}{\sqrt{2}\pi}$ (c) $\frac{2h}{\pi}$ | |
| 47. (a) $7 \times 10^6 \text{ m}^{-1}$ (b) $R_H = R_H Z^2$ | | |
| 48. 1221 Å | 49. $10 \times 10^{-8} \text{ m}$ | 50. $87.16 \times 10^{-19} \text{ J}$ |
| 51. 3.235 | 52. $1.598 \times 10^{-4} \text{ m/sec.}$ | 53. 6.4×10^{24} |
| 54. Total spin = $\pm 3/2, \pm 3, \pm 1, 0$: magnetic moment = $\sqrt{15}, \sqrt{48}, \sqrt{8}, 0$ | | 55. 26 |
| 56. $8 \times 10^{15} \text{ Hz}$ | 57. $13.297 \times 10^{-10} \text{ m}$ | 58. 184.2 kJ |

EXERCISE (Level-4)

SECTION-A

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (C) | 2. (D) | 3. (C) | 4. (B) | 5. (B) | 6. (C) | 7. (C) |
| 8. (B) | 9. (C) | 10. (A) | 11. (A) | 12. (C) | 13. (B) | 14. (B) |
| 15. (B) | 16. (A) | 17. (C) | 18. (A) | 19. (B) | 20. (B) | 21. (C) |
| 22. (D) | 23. (B) | 24. (D) | 25. (D) | 26. (A) | | |

SECTION-B

- | | | | | | | |
|---|--|--|---------|---------|---------|--------|
| 1. (A) | 2. (a) 2.18×10^8 cm/sec (b) 3.3 \AA (C) $\sqrt{2} \frac{h}{2\pi}$ | 3. $a \rightarrow R$; $b \rightarrow Q$; $c \rightarrow P$; $d \rightarrow S$ | | | | |
| 4. $a \rightarrow Q, R$; $b \rightarrow P, Q, R, S$; $c \rightarrow P, Q, R$; $d \rightarrow P, Q$ | 5.(B) | 6.(C) | 7.(B) | | | |
| 8.[9] | 9.[4] | 10. (C) | 11. [6] | 12. [3] | 13. (A) | 14.(B) |
| 15.(C) | 16.(A) | | | | | |

EXERCISE (Level-5)

- | | | | | |
|--|---|-----------------------|----------|---------------------|
| 1. $V = 2\%$ | 2. $\frac{20}{63}$ | 3. 2.45 \AA | 4. 0.33 | 5. $n \approx 6$ |
| 6. $r_1 = \frac{a_0}{2Z}, \frac{a_0}{Z}, \frac{3a_0}{Z}$ | 7. $v = \frac{n^3 h^3}{24 K e^2 \pi^3 m^2}$ | 8. 2 | 9. 26.9% | 10. $\frac{32R}{9}$ |

FUNCTION

JEE ADVANCED SYLLABUS

1. *Domain and range of functions*
2. *Into, Onto and one-to-one function*
3. *Sum, Difference, Product and quotient of two functions*
4. *Composite Function*
5. *Absolute value*
6. *Greatest integer, Polynomial, Rational, Trigonometric, Exponential and logarithmic functions*
7. *Even and odd functions*
8. *Inverse of a function*

Revision Plan

Prepare Your Revision plan today!

After attempting Exercise Sheet, please fill below table as per the instruction given.

- A. Write Question Number (QN) which you are unable to solve at your own in **column A**.
- B. After discussing the Questions written in **column A** with faculty, strike off them in the manner so that you can see at the time question number during Revision, to solve such questions again.
- C. Write down the Question Number you feel are important or good in the **column B**.

EXERCISE	COLUMN A	COLUMN B
	Questions unable to solve in first attempt	Good or Important questions
Level-1		
Level-2		
Level-3		
Level-4		
Level-5		

Revision Strategy:

Whenever you wish to revision this chapter, follow the following steps-

Step-1: Review your theory notes.

Step-2: Solve Questions of Column A

Step-3: Solve Questions of Column B

Step-4: Solve questions from other Question Bank, Problem book etc.

Function

KEY CONCEPT

1. Numbers and their sets

- (i) Natural Numbers : $N = \{1, 2, 3, 4, \dots\}$
- (ii) Whole Numbers : $W = \{0, 1, 2, 3, 4, \dots\}$
- (iii) Integer Numbers :
 I or $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
 $Z^+ = \{1, 2, 3, \dots\}$
 $Z^- = \{-1, -2, -3, \dots\}$
 $Z_0 = \{\pm 1, \pm 2, \pm 3, \dots\}$
- (iv) Rational Numbers: $Q = \left\{ \frac{p}{q} ; p, q \in Z, q \neq 0 \right\}$
- (v) Irrational numbers : The numbers which are not rational or which can not be written in the form of p/q , called irrational numbers
eg. $\{\sqrt{2}, \sqrt{3}, 2^{1/3}, 5^{1/4}, \pi, e, \dots\}$

2. Tricotomy Law

The real numbers are ordered in magnitude means. If x and y be two real numbers then there will be one and only one of the following relation will hold.

$$x < y, x = y, x > y$$

3. Interval

- (i) Close interval $[a, b] = \{x, a \leq x \leq b\}$
- (ii) Open interval (a, b) or $]a, b[= \{x, a < x < b\}$
- (iii) Semi open or semi close interval
 $[a, b[$ or $[a, b) = \{x; a \leq x < b\}$
 $]a, b]$ or $(a, b] = \{x; a < x \leq b\}$

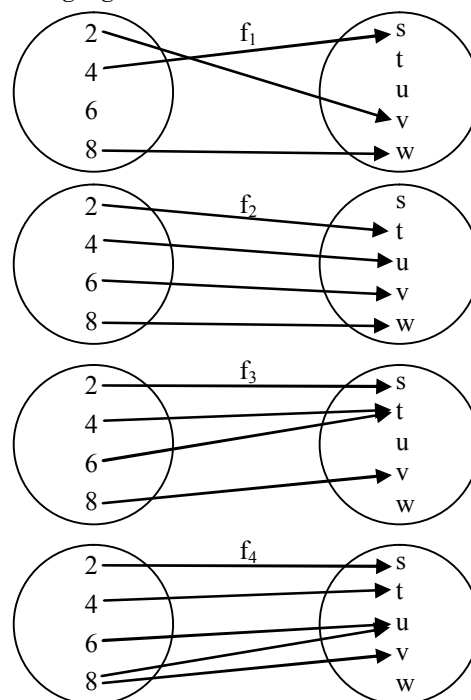
4. Function

Let A and B be two given sets and if each element $a \in A$ is associated with a unique element $b \in B$ under a rule f , then this relation is called Function.

Here, b is called the image of a and a is called the pre-image of b under f .

Example :

Let $A = \{2, 4, 6, 8\}$ and $B = \{s, t, u, v, w\}$ be two sets and let f_1, f_2, f_3 and f_4 be rules associating elements of A to elements of B as shown in the following figures.

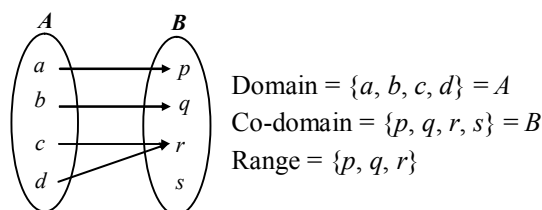


Now see that f_1 is not function from set A to set B , since there is an element $6 \in A$ which is not associated to any element of B . But f_2 and f_3 are the functions from A to B , because under f_2 and f_3 each element in A is associated to a unique element in B . But f_4 is not a function from A to B because an element $8 \in A$ is associated to two element u and w in B .

5. Domain and Range

Domain = All possible values of x for which $f(x)$ exists.

Range = For all values of x , all possible values of $f(x)$.



6. Domain and Range of some important functions

FUNCTION ($y = f(x)$)	DOMAIN (i.e. values taken by x)	RANGE (i.e. values taken by $f(x)$)
Algebraic Functions		
(i) $x^n, (n \in \mathbb{N})$	\mathbb{R} = (set of real numbers)	\mathbb{R} , if n is odd $\mathbb{R}^+ \cup \{0\}$, if n is even
(ii) $\frac{1}{x^n}, (n \in \mathbb{N})$	$\mathbb{R} - \{0\}$	$\mathbb{R} - \{0\}$, if n is odd \mathbb{R}^+ , if n is even
(iii) $x^{1/n}, (n \in \mathbb{N})$	\mathbb{R} , if n is odd $\mathbb{R}^+ \cup \{0\}$, if n is even	\mathbb{R} , if n is odd $\mathbb{R}^+ \cup \{0\}$, if n is even
(iv) $\frac{1}{x^{1/n}}, (n \in \mathbb{N})$	$\mathbb{R} - \{0\}$, if n is odd \mathbb{R}^+ , if n is even	$\mathbb{R} - \{0\}$, if n is odd \mathbb{R}^+ , if n is even
Trigonometric Functions		
(i) $\sin x$	\mathbb{R}	$[-1, 1]$
(ii) $\cos x$	\mathbb{R}	$[-1, 1]$
(iii) $\tan x$	$\mathbb{R} - \left\{(2k+1)\frac{\pi}{2}\right\}, k \in \mathbb{I}$	\mathbb{R}
(iv) $\sec x$	$\mathbb{R} - \left\{(2k+1)\frac{\pi}{2}\right\}, k \in \mathbb{I}$	$(-\infty, -1] \cup [1, \infty)$
(v) $\operatorname{cosec} x$	$\mathbb{R} - \{k\pi\}, k \in \mathbb{I}$	$(-\infty, -1] \cup [1, \infty)$
(vi) $\cot x$	$\mathbb{R} - \{k\pi\}, k \in \mathbb{I}$	\mathbb{R}
Inverse Circular Functions		
(i) $\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(ii) $\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
(iii) $\tan^{-1} x$	\mathbb{R}	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(iv) $\operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
(v) $\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
(vi) $\cot^{-1} x$	\mathbb{R}	$(0, \pi)$
Exponential Functions		
(i) e^x	\mathbb{R}	\mathbb{R}^+
(ii) $e^{1/x}$	$\mathbb{R} - \{0\}$	$\mathbb{R}^+ - \{1\}$

(iii) $a^x, a > 0$	R	R^+
(iv) $a^{1/x}, a > 0$	$R - \{0\}$	$R^+ - \{1\}$
Logarithmic Functions		
(i) $\log_a x, (a > 0) (a \neq 1)$	R^+	R
(ii) $\log_x a = \frac{1}{\log_a x} (a > 0) (a \neq 1)$	$R^+ - \{1\}$	$R - \{0\}$
Integral part Functions		
(i) $[x]$	R	I
(ii) $\frac{1}{[x]}$	$R - [0, 1)$	$\left\{\frac{1}{n}, n \in I - \{0\}\right\}$
Fractional part Functions		
(i) $\{x\}$	R	$[0, 1)$
(ii) $\frac{1}{\{x\}}$	$R - I$	$(1, \infty)$
Modulus Functions		
(i) $ x $	R	$R^+ \cup \{0\}$
(ii) $\frac{1}{ x }$	$R - \{0\}$	R^+
Signum Function		
$\text{sgn}(x) = \begin{cases} \frac{ x }{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$	R	$\{-1, 0, 1\}$
Constant Function		
$f(x) = c$	R	$\{c\}$

7. Rules of domain

$$\text{Dom}(f + g + h \dots) = \text{Dom } f \cap \text{Dom } g \cap \text{Dom } h \dots$$

$$\text{Dom}(f - g) = \text{Dom } f \cap \text{Dom } g$$

$$\text{Dom}(f \times g \times h \dots) = \text{Dom } f \cap \text{Dom } g \cap \text{Dom } h \dots$$

$$\text{Dom}\left(\frac{f}{g}\right) = \text{Dom } f \cap \text{Dom } g - \{x : g(x) = 0\}$$

8. Kinds of Mapping

(i) **One-one Function or Injection:** A function $f : A \rightarrow B$ is said to be one-one if different elements of A have different images in B .

(ii) **Many-one Function:** A function $f : A \rightarrow B$ is called many-one, if two or more different elements of A have the same f -image in B .

(iii) **Onto Function or Surjection:** A function $f : A \rightarrow B$ is onto if the each element of B has its pre-image in A .

In other words, range of $f = \text{Co-domain of } f$

(iv) **Into Function:** A function $f : A \rightarrow B$ is into if there exist at least one element in B which is not the f -image of any element in A .

In other words, range of $f \neq \text{co-domain of } f$

(v) **One-one onto Function or bijection :** A function f is said to be one-one onto if f is one-one and onto both.

(vi) **One-one into Function:** A function is said to be one-one into if f is one-one but not onto.

(vii) Many one-onto Function: A function $f : A \rightarrow B$ is many one-onto if f is onto but not one-one.

(i) $f : R \rightarrow R^+ \cup \{0\}$, $f(x) = x^2$.

(ii) $f : R \rightarrow [0, \infty)$, $f(x) = |x|$

(viii) Many one-into Function: A function f is said to be many one-into if it is neither one-one nor onto.

(i) $f : R \rightarrow R$, $f(x) = \sin x$

(ii) $f : R \rightarrow R$, $f(x) = |x|$

(ix) Identity Function: Let A be any set and the function $f : A \rightarrow A$ be defined as $f(x) = x$, $\forall x \in A$ i.e. if each element of A is mapped by itself then f is called the identity function. It is represented by I_A . If $A = \{x, y, z\}$ then $I_A = \{(x, x), (y, y), (z, z)\}$.

9. Even and Odd function

Even function

If we put $(-x)$ in place of x in the given function and if $f(-x) = f(x)$, $\forall x \in \text{domain}$ then function $f(x)$ is called even function.

Odd function

If we put $(-x)$ in place of x in the given function and if $f(-x) = -f(x)$, $\forall x \in \text{domain}$ then $f(x)$ is called odd function.

Properties of even and odd Function

- (i) The product of two even functions is even function.
- (ii) The sum and difference of two even functions is even function.
- (iii) The sum and difference of two odd functions is odd function.
- (iv) The product of two odd functions is even function.
- (v) The product of an even and an odd function is odd function.
- (vi) The sum of even and odd function is neither even nor odd function.

10. Explicit and Implicit Function

Explicit Function

A function is said to be explicit if it can be expressed directly in terms of the independent variable. $y = f(x)$ or $x = \phi(y)$

Implicit Function

A function is said to be implicit if it can not be expressed directly in terms of the independent variable.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

11. Increasing and Decreasing Function

Increasing Function

A function $f(x)$ is called increasing function in the domain D if the value of the function does not decrease by increasing the value of x . If $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ or $f'(x) > 0$ for increasing and $f'(x) \geq 0$ for not decreasing.

Decreasing Function

A function $f(x)$ is said to be decreasing function in the domain D if the value of the function does not increase by increasing the value of x (variable). If $x_1 > x_2$

$$\Rightarrow f(x_1) < f(x_2) \text{ or } x_1 < x_2$$

$\Rightarrow f(x_1) > f(x_2)$ or $f'(x) > 0$ for decreasing and $f'(x) \leq 0$ for not decreasing.

12. Greatest Integer Function

A function is said to be greatest integer function if it is of the form of $f(x) = [x]$ where $[x]$ = integer equal or less than x . i.e. $[4.2] = 4$, $[-4.4] = -5$

Properties of G.I.F :

- (i) $[x] = x$ if x is integer
- (ii) $[x + I] = [x] + I$, if I is an integer
- (iii) $[x + y] \geq [x] + [y]$
- (iv) If $[\phi(x)] \geq I$ then $\phi(x) \geq I$
- (v) If $[\phi(x)] \leq I$ then $\phi(x) < I + 1$
- (vi) If $[x] > n \Rightarrow x \geq n + 1$

- (vii) If $[x] < n \Rightarrow x < n, n \in \mathbb{I}$
 (viii) $[-x] = -[x]$ if $\forall x \in \mathbb{I}$
 (ix) $[-x] = -[x] - 1$ if $x \notin \text{Integer}$
 (x) $[x + y] = [x] + [y + x - [x]] \forall x, y \in \mathbb{R}$
 (xi) $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \dots + \left[x + \frac{n-1}{n}\right]$
 $= [nx]; n \in \mathbb{N}$

13. Fractional Part Function

It is denoted as $f(x) = \{x\}$ and defined as

- (i) $\{x\} = f$ if $x = n + f$ where $n \in \mathbb{I}$ and $0 \leq f < 1$
 (ii) $\{x\} = x - [x]$

Keep in mind \rightarrow For proper fraction $0 < f < 1$,

14. Signum Function

The signum function f is defined as

$$\text{Sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

15. Modulus Function

It is given by $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Properties of Modulus function:

- (i) $|x| \leq a \Rightarrow -a \leq x \leq a$
 (ii) $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$
 (iii) $|x + y| = |x| + |y|$
 $\Rightarrow x, y \geq 0$ or $x \leq 0, y \leq 0$
 (iv) $|x - y| = |x| - |y| \Rightarrow x \geq 0$ & $|x| \geq |y|$ or $x \leq 0$ and $y \leq 0$ and $|x| \geq |y|$
 (v) $|x \pm y| \leq |x| + |y|$
 (vi) $|x \pm y| \geq |x| - |y|$

16. Periodic Function

A function is said to be periodic function if its each value is repeated after a definite interval. So a function $f(x)$ will be periodic if a positive real number T exist such that,

$$f(x + T) = f(x), \forall x \in \text{Domain}$$

Here the least positive value of T is called the period of the function.

For example, $\sin x, \cos x, \tan x$ are periodic functions with period $2\pi, 2\pi$ & π respectively.

Note:

- (i) If function $f(x)$ has period T then

(a) $f(nx)$ has period $\frac{T}{n}$

(b) $f\left(\frac{x}{n}\right)$ has period nT

(c) $f(ax + b)$ has period $\frac{T}{|a|}$

- (ii) If the period of $f(x)$ and $g(x)$ are same (T) then the period of $af(x) + bg(x)$ will also be T .
 (iii) If the period of $f(x)$ is T_1 and $g(x)$ has T_2 , then the period of $f(x) \pm g(x)$ will be LCM of T_1 and T_2 provided it satisfies the definition of periodic function.

17. Inverse Function

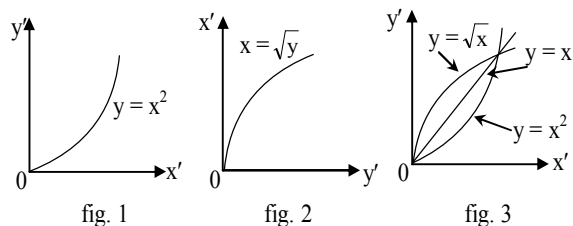
If $f: A \rightarrow B$ be a one-one onto (bijection) function, then the mapping $f^{-1}: B \rightarrow A$ which associates each element $b \in B$ with element $a \in A$, such that $f(a) = b$, is called the inverse function of the function $f: A \rightarrow B$

$$f^{-1}: B \rightarrow A, f^{-1}(b) = a \Rightarrow f(a) = b$$

Note : For the existence of inverse function, it should be one-one and onto.

Properties of Inverse Function

- (i) The inverse of a bijection is unique.
 (ii) If $f: A \rightarrow B$ is a bijection & $g: B \rightarrow A$ is the inverse of f , then $f \circ g = I_B$ and $g \circ f = I_A$, where I_A and I_B are identity functions on the sets A and B respectively. Note that the graphs of f and g are the mirror images of each other in the line $y = x$. As shown in the figure given below a point (x', y') corresponding to $y = x^2$ ($x \geq 0$) changes to (y', x') corresponding to $y = +\sqrt{x}$, the changed form of $x = \sqrt{y}$.



- (iii) The inverse of a bijection is also a bijection.
- (iv) If f & g are two bijections $f : A \rightarrow B$, $g : B \rightarrow C$ then the inverse of $g \circ f$ exists & $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

18. Composite Function

If $f : A \rightarrow B$ and $g : B \rightarrow C$ are two function then the composite function of f and g , $g \circ f : A \rightarrow C$ will be defined as $g \circ f(x) = g[f(x)]$, $\forall x \in A$.

Properties of Composite Function

- If f and g are two functions then for composite of two functions $f \circ g \neq g \circ f$.
- Composite functions obeys the property of associativity i.e. $f \circ (g \circ h) = (f \circ g) \circ h$.
- Composite function of two one-one onto functions if exist, will also be a one-one onto function.

Algebra of function

- $(f \circ g)(x) = f[g(x)]$
- $(f \circ f)(x) = f[f(x)]$
- $(g \circ g)(x) = g[g(x)]$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $(f \pm g)(x) = f(x) \pm g(x)$
- $(f/g)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

- * Composite functions in not commutative
- * Let f and g are two functions then if f & g are injective or surjective or bijective then " $g \circ f$ " also injective or surjective or bijective.

19. Even and Odd function

Even Extension:

If a function $f(x)$ is defined on the interval $[0, a]$, $0 \leq x \leq a \Rightarrow -a \leq -x \leq 0$ we define $f(x)$ in the $[-a, 0]$ such that $f(x) = f(-x)$.

$$\text{Let } I(x) = \begin{cases} f(x) & : x \in [0, a] \\ f(-x) & : x \in [-a, 0] \end{cases}$$

Odd Extension:

If a function $f(x)$ is defined on the interval $[0, a]$, $0 \leq x \leq a \Rightarrow -a \leq -x \leq 0$

$\therefore x \in [-a, 0]$, we define $f(x) = -f(-x)$. Let I be the odd extension then

$$I(x) = \begin{cases} f(x), & x \in [0, a] \\ -f(-x), & x \in [-a, 0] \end{cases}$$

20. Some very important point

(a) If x, y are independent variables then :

- If $f(x, y) = f(x) + f(y)$ then $\Rightarrow f(x) = k \log x$
- If $f(xy) = f(x) \cdot f(y)$ then $\Rightarrow f(x) = x^n$, $n \in R$
- If $f(x+y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$
- If $f(x+y) = f(x) + f(y) \Rightarrow f(x) = x$
- If $f(x+y) = f(x) = f(y) \Rightarrow f(x) = k$, here k is constant
- By considering a general n^{th} degree polynomial and writing the expression

$$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \Rightarrow f(x) = \pm x^n + 1$$

(b) Algebraic Functions

(i) Polynomial function:

A function having the form

$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots a_nx^n$, where $a_0, a_1, a_2, \dots, a_n$ are real constant, $a_n \neq 0$ and $n \in N$ called rational integral function or polynomial of degree n .

- Rational Function:** The ratio of two polynomial is called Fraction Rational function or simply rational function.

$$\text{e.g. } y = \frac{x^{12} + x^2 - 1}{x^6 + x^4 + 1}$$

- Irrational Function:** Functions with operations of addition, subtraction, multiplication, division and raising to power with non-integral rational exponent are called irrational functions.

$$(I) y = \sqrt{x} \quad (II) y = \frac{\sqrt{x^3 + 1} - \sqrt{x^{11}}}{\sqrt{x^2 + x + 1}}$$

Such type of function are called Irrational function.

(iv) **Transcendental function** : All those function who has infinite terms while expanded are called transdental function. for example all trigonometrical function.

Inverse trigonometrical function, exponential function, logarithmic function etc.

e.g. $f(x) = \sin x$, $y = \cos^{-1} x$

$y = \log_e x$, $y = \sqrt{\log_e x - \sin^{-1} x}$

(c) Mapping :

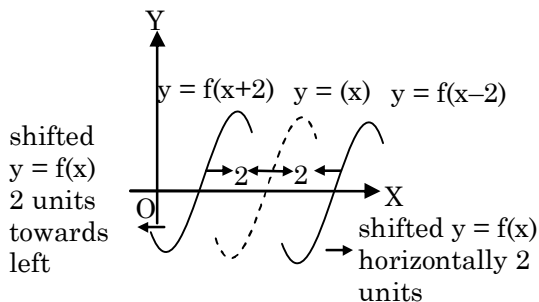
One-one or injective mapping or monomorphism. If $f : A \rightarrow B$ is one-one mapping A has m element and B has n element hence the no. of mappings

$$= \begin{cases} {}^n P_m, & n \geq m \\ 0, & n < m \end{cases}$$

(d) If graph of $y = f(x)$ be known then to find the graph of

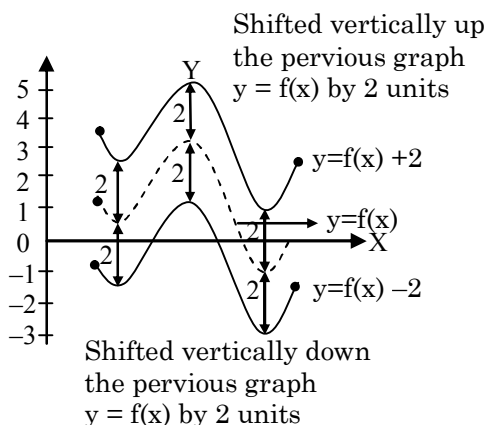
(i) $y = f(x - a)$ or $y = f(x + a)$

To find $y = f(x - a)$ (Let $a = 2$)

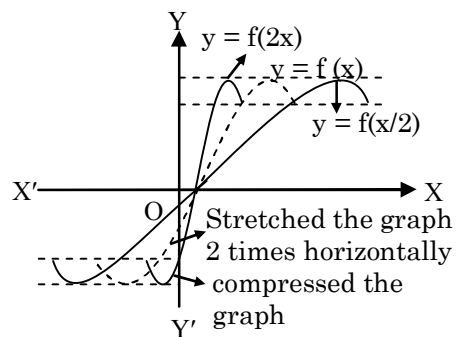


(ii) $y = f(x) + a$: or $y = f(x) - a$

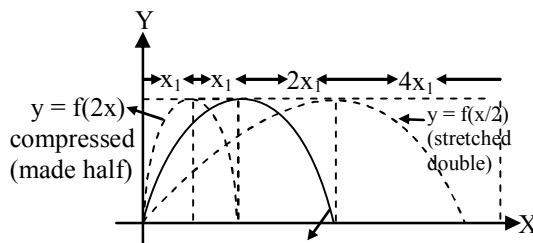
(Let $a = 2$)



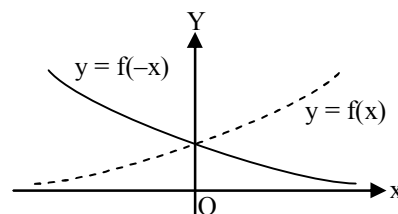
(iii) $y = f(x/a)$ or $y = f(ax)$: (Let $a = 2, 1/2$)



See more examples about the same.



(iv) $y = f(-x)$:

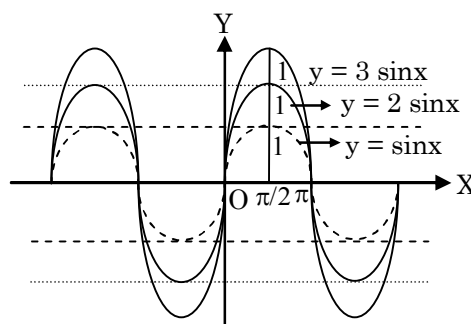


Reflection of $y = f(x)$ w. r. t. axis of y is $y = f(-x)$

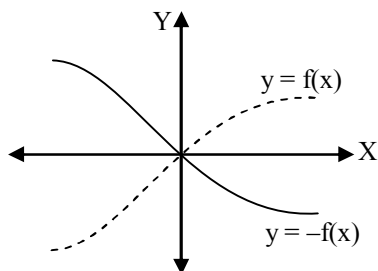
(v) To find $y = k f(x)$:

Rule – Stretch the previous graph k times vertically

e.g. see below $y = 2 \sin x$, $y = 3 \sin x$

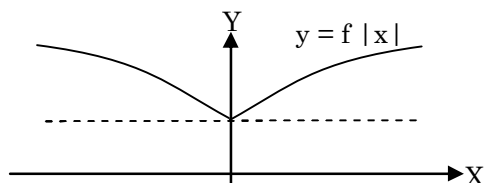
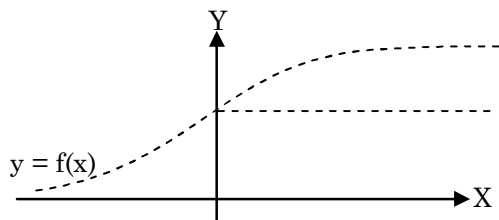


(vi) $y = -f(x)$:



Reflection of $y = f(x)$ w.r.t. axis of x is $y = -f(x)$

(vii) To find $y = f|x|$:

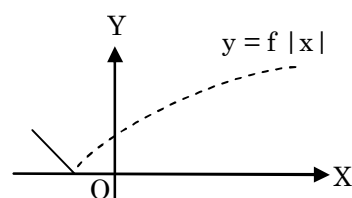
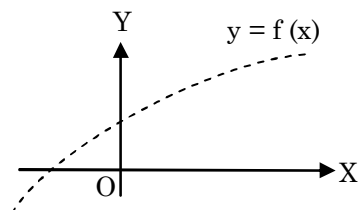


Rule : Neglect the graph lying in IInd and IIIrd quadrant and, Take the image of graph lying in I and IVth quadrant w. r. t. axis of y .

The original graph including its image is called $y = f|x|$.

Here we took the image of the portion lying in first quadrant about axis of y and left the portion which was lying in second quadrant.

(viii) To find $y = |f(x)|$:



Rule : Take the image of the portion line below axis of x about axis of x . Remain as it is the portion above the axis of x .

SOLVED EXAMPLES

Ex.1 Find the domain and range of the function

$$f(x) = \sqrt{2-x} + \sqrt{1+x}$$

Sol. Domain of $f(x) = \{x \mid 2-x \geq 0 \text{ and } 1+x \geq 0\}$

$$\therefore \text{domain of } f(x) = [-1, 2]$$

$$\begin{aligned} \text{Again, } \{f(x)\}^2 &= (\sqrt{2-x} + \sqrt{1+x})^2 \\ &= 3 + 2\sqrt{(2-x)(1+x)} \\ &= 3 + 2\sqrt{2+x-x^2} \\ &= 3 + 2\sqrt{\frac{9}{4} - \left(x - \frac{1}{2}\right)^2} \end{aligned}$$

\therefore the greatest value of $\{f(x)\}^2$

$$= 3 + 2\sqrt{\frac{9}{4}} = 6, \text{ when } x = \frac{1}{2}$$

the least value of $\{f(x)\}^2 = 3 + 0 = 3$,

$$\text{when } x - \frac{1}{2} = \frac{3}{2}, \text{ i.e. } x = 2$$

\therefore the greatest value of $f(x) = \sqrt{6}$

and the least value of $f(x) = \sqrt{3}$

\therefore range of $f(x) = [\sqrt{3}, \sqrt{6}]$

Ex.2 Find the range of the following function

$$f(x) = \frac{3}{2-x^2}$$

Sol. Let $y = \frac{3}{2-x^2} = f(x) \quad \dots(1)$

The function y is not defined for

$$x = \pm \sqrt{2}$$

$$\text{From (1), } x^2 = \frac{2y-3}{y}$$

since for real x , $x^2 \geq 0$,

$$\text{We have } \frac{2y-3}{y} \geq 0$$

$\therefore y \geq 3/2$ or $y < 0$ (Note that $y \neq 0$)

Hence the range of the function is

$$]-\infty, 0[\cup [3/2, \infty)$$

Ex.3 Find the range of the following function:

$$f(x) = \log_2 \left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}} \right)$$

Sol. $\therefore f(x) = \log_2 \left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}} \right)$

$$= \log_2 \left(\sin \left(\pi - \frac{\pi}{4} \right) + 3 \right) = y \text{ (let)}$$

$$\Rightarrow 2^y = \sin \left(\pi - \frac{\pi}{4} \right) + 3$$

$$\Rightarrow 2^y - 3 = \sin \left(\pi - \frac{\pi}{4} \right)$$

$$\text{But } -1 \leq \sin \left(\pi - \frac{\pi}{4} \right) \leq 1$$

$$\therefore -1 \leq 2^y - 3 \leq 1$$

$$\Rightarrow 2 \leq 2^y \leq 4$$

$$\Rightarrow 2^1 \leq 2^y \leq 2^2$$

Hence $y \in [1, 2]$.

Hence Range of $f(x)$ is $[1, 2]$.

Ex.4 Find the period of the following function

$$f(x) = e^{x-[x]} + |\cos \pi x| + |\cos 2\pi x| + \dots + |\cos n\pi x|,$$

[.] is greatest integer function.

Sol. $f(x) = e^{x-[x]} + |\cos \pi x| + |\cos 2\pi x| + \dots + |\cos n\pi x|$

Period of $x - [x] = 1$

Period of $|\cos \pi x| = 1$

Period of $|\cos 2\pi x| = 1/2$

.....

.....

Period of $|\cos n\pi x| = 1/n$

So period of $f(x)$ will be

L.C.M. of all periods = 1.

Ex.5 Let a function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x - [x]$, (where $[x]$ is a greatest integer $\leq x$), for all $x \in \mathbb{R}$. Is the function bijective?

Sol. Let $x_1, x_2 \in \mathbb{R}$

$$f(x_1) = f(x_2) \Rightarrow x_1 - [x_1] = x_2 - [x_2]$$

$$\Rightarrow x_1 \neq x_2$$

\therefore The function is not bijective.

Ex.6 If $f(x) = \begin{cases} x^3 + 1, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$,

$$g(x) = \begin{cases} (x-1)^{1/3}, & x < 1 \\ (x-1)^{1/2}, & x \geq 1 \end{cases}$$

Compute $g \circ f(x)$.

Sol. We have

$$g \circ f(x) = g(f(x))$$

Ex.7 The value of $n \in \mathbb{I}$ for which the function

$$f(x) = \frac{\sin nx}{\sin(x/n)} \text{ has } 4\pi \text{ as its period is -}$$

(A) 2 (B) 3 (C) 5 (D) 4

Sol. For $n = 2$,

$$\text{we have } \frac{\sin 2x}{\sin(x/2)} = 4(\cos x/2) \cos x.$$

The period of $\cos x$ is 2π , & that of $\cos(x/2)$ is 4π .

Hence the period of $\frac{\sin 2x}{\sin(x/2)}$ is 4π .

Also, the period of $\frac{\sin 3x}{\sin(x/3)}$, $\frac{\sin 5x}{\sin(x/5)}$

and $\frac{\sin 4x}{\sin(x/4)}$ cannot be 4π . **Ans. [A]**

Ex.8 Prove that even functions do not have inverse.

Sol. Even functions are many one function and for the existence of inverse function should be one-one. Hence inverse of an even function will not exist.

Ex.9 Prove that periodic functions do not have inverse.

Sol. $f(x)$ is periodic

$\Rightarrow f$ is many one

$\Rightarrow f^{-1}$ does not exist.

$$= \begin{cases} g(x^3 + 1), & x < 0 \\ g(x^2 + 1), & x \geq 0 \end{cases}$$

$$= \begin{cases} (x^3 + 1 - 1)^{1/3} & x < 0 \\ (x^2 + 1 - 1)^{1/2}, & x \geq 0 \end{cases}$$

$$= \begin{cases} x, & x < 0 \\ x, & x \geq 0 \end{cases} = x \text{ for all } x.$$

Hence, $gof(x) = x$ for all x .

Ex.10 Show that the function $f: R \rightarrow R$ defined by $f(x) = 3x^3 + 5$ for all $x \in R$ is a bijection.

Sol. Injectivity : Let x, y be any two elements of $R(\text{domain})$.

Then,

$$f(x) = f(y) \Rightarrow 3x^3 + 5 = 3y^3 + 5$$

$$\Rightarrow x^3 = y^3 \Rightarrow x = y$$

Thus, $f(x) = f(y)$

$\Rightarrow x = y$ for all $x, y \in R$.

so, f is an injective map.

Surjectivity : Let y be an arbitrary element of $R(\text{co-domain})$.

Then,

$$f(x) = y \Rightarrow 3x^3 + 5 = y \Rightarrow x^3 = \frac{y-5}{3}$$

$$\Rightarrow x = \left(\frac{y-5}{3} \right)^{1/3}$$

Thus we find that for all $y \in R$ (co-domain)

there exists $x = \left(\frac{y-5}{3} \right)^{1/3} \in R$ (domain)

such that

$$f(x) = f\left(\left(\frac{y-5}{3}\right)^{1/3}\right) = 3\left[\left(\frac{y-5}{3}\right)^{1/3}\right]^3 + 5$$

$$= y - 5 + 5 = y$$

This shows that every element in the co-domain has its pre-image in the domain. So, f is a surjection. Hence, f is a bijection.

Ex.11 Let, $f(x) = x + 1$, $x \leq 1$

$$= 2x + 1, \quad 1 < x \leq 2$$

$$g(x) = x^2, \quad -1 \leq x < 2$$

$$= x + 2, \quad 2 \leq x \leq 3$$

Find fog and gof .

Sol. $f\{g(x)\} = g(x) + 1, \quad g(x) \leq 1$
 $= 2g(x) + 1, \quad 1 < g(x) \leq 2$

$$\Rightarrow f\{g(x)\} = x^2 + 1, \quad -1 \leq x \leq 1$$

$$= 2x^2 + 1, \quad 1 < x \leq \sqrt{2}$$

$$g\{f(x)\} = \{f(x)\}^2, \quad -1 \leq f(x) < 2$$

$$= f(x) + 2, \quad 2 \leq f(x) \leq 3$$

$$gof(x) = (x + 1)^2, \quad -2 \leq x < 1$$

$$= (x + 1)^2, \quad -2 \leq x \leq 1$$

Ex.12 Find the inverse of the following function :

$$f(x) = \begin{cases} x, & x < 1 \\ x^2, & 1 \leq x \leq 4 \\ 8\sqrt{x}, & x > 4 \end{cases}$$

Sol. Let $f(x) = \begin{cases} x, & x < 1 \\ x^2, & 1 \leq x \leq 4 \\ 8\sqrt{x}, & x > 4 \end{cases}$

Let $f(x) = y \quad \therefore x = f^{-1}(y)$

$$\Rightarrow x = \begin{cases} y, & y < 1 \\ \sqrt{y}, & 1 \leq y \leq 16 \\ y^2/64, & y > 16 \end{cases}$$

$$\Rightarrow f^{-1}(y) = \begin{cases} y, & y < 1 \\ \sqrt{y}, & 1 \leq y \leq 16 \\ y^2/64, & y > 16 \end{cases}$$

$$\Rightarrow f^{-1}(x) = \begin{cases} x, & x < 1 \\ \sqrt{x}, & 1 \leq x \leq 16 \\ x^2/64, & x > 16 \end{cases}$$

Ex.13 Let $f(x) = x^2 + x$ be defined on the interval $[0, 2]$. Find the odd and even extensions of $f(x)$ in the interval $[-2, 2]$.

Sol. Odd extension.

$$f(x) = \begin{cases} f(x), & 0 \leq x \leq 2 \\ -f(-x), & -2 \leq x < 0 \end{cases}$$

$$= \begin{cases} x^2 + x, & 0 \leq x \leq 2 \\ -x^2 + x, & -2 \leq x < 0 \end{cases}$$

Even extension

$$f(x) = \begin{cases} f(x), & 0 \leq x \leq 2 \\ f(-x), & -2 \leq x < 0 \end{cases}$$

$$= \begin{cases} x^2 + x, & 0 \leq x \leq 2 \\ x^2 - x, & -2 \leq x < 0 \end{cases}$$

Ex.14 Let $f: R \rightarrow R$ be given by

$f(x) = (x+1)^2 - 1, x \geq -1$. Show that f is invertible. Also, find the set $S = \{x : f(x) = f^{-1}(x)\}$.

Sol. In order to show that $f(x)$ is invertible, it is sufficient to show that $f(x)$ is a bijection.

f is an injection : For any $x, y \in R$ satisfying $x \geq -1, y \geq -1$,

We have $f(x) = f(y)$

$$\Rightarrow (x+1)^2 - 1 = (y+1)^2 - 1$$

$$\Rightarrow x^2 + 2x = y^2 + 2y$$

$$\Rightarrow x^2 - y^2 = -2(x - y)$$

$$\Rightarrow (x - y)(x + y) = -2(x - y)$$

$$\Rightarrow (x - y)[x + y + 2] = 0$$

$$\Rightarrow x - y = 0 \text{ or } x + y + 2 = 0$$

$$\Rightarrow x = y \text{ or } x = y = -1$$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all

$$x \geq -1, y \geq -1.$$

So, $f(x)$ is an injection.

f is a surjection : For all $y \geq -1$ there exists.

$$x = -1 + \sqrt{y+1} \geq -1 \text{ such that } f(x) = y$$

So, $f(x)$ is a surjection.

Hence, f is a bijection. Consequently, it is invertible.

$$f(x) = f^{-1}(x) \Rightarrow f(x) = x$$

$$(x+1)^2 - 1 = x \Rightarrow x = 0, -1$$

Ex.15 If f, g, h are function from R to R such that

$$f(x) = x^2 - 1, g(x) = \sqrt{x^2 + 1},$$

$$h(x) = 0, \text{ if } x \leq 0$$

$$= x, \text{ if } x \geq 0$$

then find the composite function $ho(fog)$ and determine whether the function fog is invertible and the function h is the identity function.

Sol. Here $f(x) = x^2 - 1$ for all x

$$\text{and } g(x) = \sqrt{x^2 + 1} \text{ for all } x$$

$$\therefore f\{g(x)\} = \{g(x)\}^2 - 1$$

$$= x^2 + 1 - 1 = x^2 \text{ for all } x$$

$$\therefore h\{f(g(x))\} = h(x^2) = x^2$$

because $x^2 \geq 0$ [from definition of $h(x)$.]

Now, $f\{g(x)\} = x^2$ for all x

As $x^2 \geq 0$, $(fog)(x)$ cannot be negative.

So fog is not an onto function.

Hence fog is not invertible.

Again, $h(x) = x$ for $x \geq 0$.

But, by definition $h(x) \neq x$ for $x < 0$.

Hence h is not the identity function.

Ex.16 Let $f(x) = \tan x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and

$$g(x) = \sqrt{1-x^2}. \text{ Determine } fog \text{ and } gof.$$

Sol. From the given domain of $f \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ we

conclude that its range $]-\infty, \infty[$ i.e. whole of R

Domain of g is $1-x^2 \geq 0$ or $x^2 - 1 \leq 0$

or $(x+1)(x-1) \leq 0$ or $-1 \leq x \leq 1$ or $[-1, 1]$

and for range of $g, y = \sqrt{1-x^2}$

since $x^2 \leq 1 \therefore y \in [0, 1]$

$$(fog)x = f(g(x)) = f\{\sqrt{1-x^2}\}$$

$$= f(t), \text{ where } t = \sqrt{1-x^2} \in [0, 1]$$

range of $g \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ which is domain of f .

$$= \tan t = \tan \sqrt{1-x^2}$$

$(g \circ f)x = g(f(x)) = g(\tan x) = g(t)$ where $t = \tan x \in \text{range of } f = R$. But R is not a subset of domain of $g = [-1, 1]$

Hence $g \circ f$ is not defined.

Ex.17 Let $f_1(x) = \frac{x}{3} + 10$ for all $x \in R$, and

$f_n(x) = f_1(f_{n-1}(x))$ for $n \geq 2$. Then find $f_n(x)$.

Sol. We have

$$f_n(x) = f_1(f_{n-1}(x)), \quad n \geq 2$$

$$\Rightarrow f_2(x) = f_1(f_1(x)) = \frac{1}{3} f_1(x) + 10$$

$$= \frac{1}{3} \left(\frac{x}{3} + 10 \right) + 10$$

$$= \frac{x}{3^2} + \frac{10}{3} + 10$$

$$f_3(x) = f_1(f_2(x))$$

$$= \frac{1}{3} f_2(x) + 10$$

$$= \frac{1}{3} \left(\frac{x}{3^2} + \frac{10}{3} + 10 \right) + 10$$

$$= \frac{x}{3^3} + \frac{10}{3^2} + \frac{10}{3} + 10$$

$$f_4(x) = f_1(f_3(x)) = \frac{1}{3} f_3(x) + 10$$

$$= \frac{1}{3} \left(\frac{x}{3^3} + \frac{10}{3^2} + \frac{10}{3} + 10 \right) + 10$$

$$= \frac{x}{3^4} + \frac{10}{3^3} + \frac{10}{3^2} + \frac{10}{3} + 10$$

Continuing in this manner, we obtain

$$f_n(x) = \frac{x}{3^n} + \frac{10}{3^{n-1}} + \frac{10}{3^{n-2}} + \dots + \frac{10}{3} + 10$$

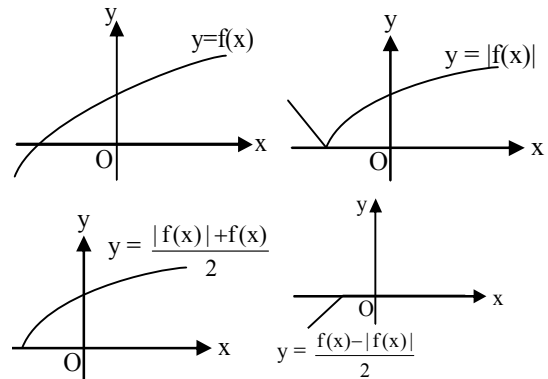
$$= \frac{x}{3^n} + 10 \left(\frac{1 - \frac{1}{3^n}}{1 - \frac{1}{3}} \right)$$

$$= \frac{x}{3^n} + 15 \left(1 - \frac{1}{3^n} \right) = \frac{x-15}{3^n} + 15$$

Ex.18 Knowing the graph of $y = f(x)$ draw

$$y = \frac{f(x) + |f(x)|}{2} \text{ and } y = \frac{f(x) - |f(x)|}{2}$$

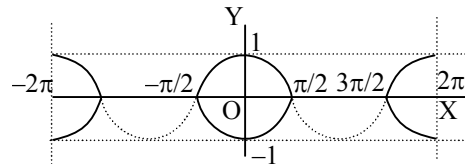
Sol. Let graph



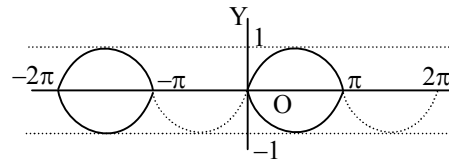
Ex.19 Draw following graphs:

(i) $|y| = \cos x$ (ii) $|y| = \sin x$

Sol. (i) $|y| = \cos x$



(ii) $|y| = \sin x$



EXERCISE (Level-1)

Question
based on

Domain

- Q.1** Domain of $y = \log_{10} \left(\frac{5x - x^2}{4} \right)$ is
 (A) (0, 5)
 (B) [1, 4]
 (C) $(-\infty, 0) \cup (5, \infty)$
 (D) $(-\infty, 1) \cup (4, \infty)$
- Q.2** The domain of definition of $f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2 + 2x + 8}}$ is:
 (A) (1, 4) (B) (-2, 4)
 (C) (2, 4) (D) [2, ∞)
- Q.3** The function $f(x) = \cot^{-1} \sqrt{(x+3)x} + \cos^{-1} \sqrt{x^2 + 3x + 1}$ is defined on the set S, where S is equal to:
 (A) {0, 3} (B) (0, 3)
 (C) {-3, 0} (D) [-3, 0]
- Q.4** The domain of $\sqrt{\sec^{-1} \left(\frac{2 - |x|}{4} \right)}$ is
 (A) R (B) $R - (-1, 1)$
 (C) $R - (-3, 3)$ (D) $R - (-6, 6)$
- Q.5** The domain of the function $f(x) = {}^{24-x}C_{3x-1} + {}^{40-6x}C_{8x-10}$ is -
 (A) {2, 3} (B) {1, 2, 3}
 (C) {1, 2, 3, 4} (D) None of these
- Q.6** Domain of the function $f(x) = (1 - 3x)^{1/3} + 3\cos^{-1} \left(\frac{2x-1}{3} \right) + 3^{3\tan^{-1}x}$ is
 (A) $\left[-\frac{1}{3}, \frac{1}{3} \right]$ (B) $\left[-\frac{1}{2}, 1 \right]$
 (C) [-1, 2] (D) $\left[-\frac{1}{4}, \frac{1}{2} \right]$

- Q.7** The function $f(x) = \frac{\sec^{-1}x}{\sqrt{x-[x]}}$, where $[x]$ denotes the greatest integer less than or equal to x , is defined for all x belonging to -
 (A) R
 (B) $R - \{(-1, 1) \cup \{n : n \in Z\}\}$
 (C) $R^+ - (0, 1)$
 (D) $R^+ - \{n : n \in N\}$
- Q.8** The function $f(x) = \cos^{-1} \left(\frac{|x| - 3}{2} \right) + [\log_e(4 - x)]^{-1}$ is defined for -
 (A) $[-1, 0] \cup [1, 5]$
 (B) $[-5, -1] \cup [1, 4]$
 (C) $[-5, -1] \cup ([1, 4] - \{3\})$
 (D) $[1, 4] - \{3\}$
- Q.9** The domain of function $f(x) = \log |\log x|$ is -
 (A) (0, ∞) (B) (1, ∞)
 (C) $(0, 1) \cup (1, \infty)$ (D) $(-\infty, 1)$

- Q.10** The domain of function $f(x) = \frac{1}{\log_{10}(3-x)} + \sqrt{x+2}$ is -
 (A) [-2, 3) (B) $[-2, 3) - \{2\}$
 (C) [-3, 2] (D) $[-2, 3] - \{2\}$

Question
based on

Range

- Q.11** The range of the function $y = \frac{1}{2 - \sin 3x}$ is :
 (A) $\left(\frac{1}{3}, 1 \right)$ (B) $\left[\frac{1}{3}, 1 \right]$
 (C) $\left[\frac{1}{3}, 1 \right]$ (D) None of these
- Q.12** The value of the function $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4x + 3}$ lies in the interval -
 (A) $(-\infty, \infty) - \left\{ \frac{1}{2}, 1 \right\}$ (B) $(-\infty, \infty)$
 (C) $(-\infty, \infty) - \{1\}$ (D) None of these

- Q.13** The range of the function,
 $y = \log_{\sqrt{7}}(\sqrt{2}(\sin x - \cos x) + 5)$ is
 (A) R (B) Z
 (C) $[\log_7 4, \log_7 5]$ (D) $[2 \log_7 3, 2]$
- Q.14** Which of the following function(s) has the range $[-1, 1]$
 (A) $f(x) = \cos(2 \sin x)$
 (B) $g(x) = \cos\left(1 - \frac{1}{1+x^2}\right)$
 (C) $h(x) = \sin(\log_2 x)$
 (D) $k(x) = \tan(e^x)$
- Q.15** The range of the function $f(x) = \cos(\cos^{-1}\{x\})$ is (where $\{ \cdot \}$ denotes the fractional part function)
 (A) $[0, 1]$ (B) $[0, 1]$
 (C) $(0, 1)$ (D) $(0, 1)$
- Q.16** Let $f(x) = \frac{x - [x]}{1 - [x] + x}$, then range of $f(x)$ is-
 (where $[\cdot]$ represent greatest integer function)
 (A) $[0, 1]$ (B) $\left[0, \frac{1}{2}\right]$
 (C) $\left[\frac{1}{2}, 1\right]$ (D) $\left[0, \frac{1}{2}\right)$
- Q.17** The range of the function $y = \log_3(5 + 4x - x^2)$ is-
 (A) $(0, 2]$ (B) $(-\infty, 2]$
 (C) $(0, 9]$ (D) None of these

Question
based on

Classification of functions

- Q.18** Let $f: R \rightarrow R$ be a function defined by
 $f(x) = \frac{x^2 + 2x + 5}{x^2 + x + 1}$ is :
 (A) one-one and into
 (B) one-one and onto
 (C) many-one and onto
 (D) many-one and into
- Q.19** The function $f: [2, \infty) \rightarrow Y$ defined by
 $f(x) = x^2 - 4x + 5$ is both one-one & onto if:
 (A) $Y = R$ (B) $Y = [1, \infty)$
 (C) $Y = [4, \infty)$ (D) $Y = [5, \infty)$

- Q.20** Let $f: R \rightarrow R$ be a function defined by
 $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is :
 (A) one - one & onto (B) one - one & into
 (C) many one & onto (D) many one & into

- Q.21** Which of the following function from
 $A = \{x: -1 \leq x \leq 1\}$ to itself are bijections-
 (A) $f(x) = \frac{x}{2}$ (B) $g(x) = \sin\left(\frac{\pi x}{2}\right)$
 (C) $h(x) = |x|$ (D) $k(x) = x^2$

- Q.22** If $f: \left[\frac{\pi}{4} - \frac{1}{2}, \frac{3\pi}{4} - \frac{1}{2}\right] \rightarrow [-1, 1]$ is defined by
 $f(x) = \sin(2x + 1)$, then f is
 (A) one one into (B) many one onto
 (C) one one onto (D) many one into

- Q.23** The number of bijective functions from set
 A to itself when A contains 106 elements is
 (A) 106 (B) 106!
 (C) 106^{106} (D) $106^{106} - 106!$

Question
based on

Inverse function

- Q.24** If $f(x) = x^3 - 1$ and domain of $f = \{0, 1, 2, 3\}$,
 then domain of f^{-1} is -
 (A) $\{0, 1, 2, 3\}$ (B) $\{1, 0, -7, -26\}$
 (C) $\{-1, 0, 7, 26\}$ (D) $\{0, -1, -2, -3\}$
- Q.25** The inverse of the function $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is
 (A) $\frac{1}{2} \log \frac{1+x}{1-x}$ (B) $\frac{1}{2} \log \frac{2+x}{2-x}$
 (C) $\frac{1}{2} \log \frac{1-x}{1+x}$ (D) $2 \log(1+x)$

Question
based on

Composite function

- Q.26** The function $f(x)$ is defined in $[0, 1]$ then
 the domain of definition of the function
 $f[\ell n(1 - x^2)]$ is given by :
 (A) $x \in \{0\}$
 (B) $x \in [-\sqrt{1+e} - 1] \cup [1 + \sqrt{1+e}]$
 (C) $x \in (-\infty, \infty)$
 (D) None of these

EXERCISE (Level-2)

Single correct answer type questions

- Q.1** If fundamental period of $\frac{\cos(\sin nx)}{\tan(x/n)}$ ($n \in \mathbb{N}$) is 6π then n is equal to
 (A) 3 (B) 2
 (C) 6 (D) 1

- Q.2** Let $f(x) = \sin^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right)$ and $g(x) = \sec^2 x - \tan^2 x$. The two functions are equal over the set -
 (A) ϕ
 (B) $\mathbb{R} - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\right\}$
 (C) \mathbb{R}
 (D) None of these

- Q.3** Domain and range of $\sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$ is
 (A) $[-2, 1), (-1, 1)$ (B) $(-2, 1), [-1, 1]$
 (C) $(-2, 1), \mathbb{R}$ (D) None of these

- Q.4** The range of $\sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$ where $[]$ represent greatest integer function
 (A) $\left\{\frac{\pi}{2}, \pi\right\}$ (B) $\{\pi\}$
 (C) $\left\{\frac{\pi}{2}\right\}$ (D) None of these

- Q.5** Let $f(x) = \frac{9^x}{9^x + 3}$ and $f(x) + f(1-x) = 1$ then find value of $f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + \dots + f\left(\frac{1995}{1996}\right)$ is -
 (A) 998 (B) 997
 (C) 997.5 (D) 998.5

- Q.6** If $f(x)$ be a polynomial satisfying $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65$ then $f(6) = ?$
 (A) 176 (B) 217
 (C) 289 (D) None of these

- Q.7** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{\sin([x]\pi)}{x^2 + 2x + 4}$, where $[]$ represent greatest integer function, then which one is not true -
 (A) f is periodic (B) f is even
 (C) f is many-one (D) f is onto

- Q.8** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x + \sqrt{x^2}$, then f is -
 (A) Injective (B) Surjective
 (C) Bijective (D) None of these

- Q.9** Which of the following functions are equal?
 (A) $f(x) = x, g(x) = \sqrt{x^2}$
 (B) $f(x) = \log x^2, g(x) = 2 \log x$
 (C) $f(x) = 1, g(x) = \sin^2 x + \cos^2 x$
 (D) $f(x) = \frac{x}{x}, g(x) = 1$

- Q.10** Let $f : (4, 6) \rightarrow (6, 8)$ be a function defined by $f(x) = x + \left[\frac{x}{2}\right]$, where $[]$ represent greatest integer function then $f^{-1}(i)$ is equal to -
 (A) $x - 2$ (B) $x - \left[\frac{x}{2}\right]$
 (C) $-x - 2$ (D) None of these

- Q.11** The interval for which $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$ holds -
 (A) $[0, \infty)$ (B) $[0, 3]$
 (C) $[0, 1]$ (D) $[0, 2]$

- Q.12** The function $f(x) = \sqrt{\log_{10} \cos(2\pi x)}$ exists -
 (A) for any rational x
 (B) only when x is a positive integer
 (C) only when x is fractional
 (D) for any integer value of x including zero

- Q.13** The domain of the function $\sec^{-1}[x^2 - x + 1]$, is given by (where $[]$ is greatest integer function) -
 (A) $[0, 1]$ (B) $(-\infty, 0] \cup [1, \infty)$
 (C) $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ (D) None of these

Q.14 The domain of definition of the function
 $f(x) = \frac{\cot^{-1} x}{\sqrt{\{x^2 - [x^2]\}}}$, where $[x]$ denotes the

greatest integer less than or equal to x is -

- (A) \mathbb{R}
- (B) $\mathbb{R} - \{\pm \sqrt{n} : n \in \mathbb{I}^+ \cup \{0\}\}$
- (C) $\mathbb{R} - \{0\}$
- (D) $\mathbb{R} - \{n : n \in \mathbb{I}\}$

Q.15 The domain of the definition of
 $f(x) = \log\{(\log x)^2 - 5 \log x + 6\}$ is equal to-

- (A) $(0, 10^2)$
- (B) $(10^3, \infty)$
- (C) $(10^2, 10^3)$
- (D) $(0, 10^2) \cup (10^3, \infty)$

Q.16 If $A = \left\{x : \frac{\pi}{6} \leq x \leq \frac{\pi}{3}\right\}$ and

$f(x) = \cos x - x(1+x)$ then $f(A)$ is equal to-

- (A) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$
- (B) $\left[-\frac{\pi}{3}, -\frac{\pi}{6}\right]$
- (C) $\left[\frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)\right]$
- (D) $\left[\frac{1}{2} + \frac{\pi}{3}\left(1 - \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} + \frac{\pi}{6}\left(1 - \frac{\pi}{6}\right)\right]$

Q.17 If A be the set of all triangles and B that of positive real numbers, then the mapping $f: A \rightarrow B$ given by $f(\Delta) = \text{area of } \Delta$, ($\Delta \in A$) is

- (A) one-one into mapping
- (B) one-one onto mapping
- (C) many-one into mapping
- (D) many-one onto mapping

Q.18 Let $f: \mathbb{R} \rightarrow A = \left\{y \mid 0 \leq y < \frac{\pi}{2}\right\}$ be a

function such that $f(x) = \tan^{-1}(x^2 + x + k)$, where k is a constant. The value of k for which f is an onto function, is -

- (A) 1
- (B) 0
- (C) $\frac{1}{4}$
- (D) None of these

Q.19 Which of the following functions are not injective mapping-

- (A) $f(x) = |x + 1|$, $x \in [-1, \infty)$
- (B) $g(x) = x + \frac{1}{x}$; $x \in (0, \infty)$
- (C) $h(x) = x^2 + 4x - 5$; $x \in (0, \infty)$
- (D) $k(x) = e^{-x}$; $x \in [0, \infty)$

Q.20 Let f be an injective map. with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$, such that exactly one of the following statements is correct and the remaining are false : $f(x) = 1$, $f(y) \neq 1$, $f(z) \neq 2$. The value of $f^{-1}(1)$ is -

- (A) x
- (B) y
- (C) z
- (D) None of these

Q.21 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be two one-one onto functions such that they are mirror image of each other about the line $y = 0$, then $h(x) = f(x) + g(x)$ is-

- (A) One-one and onto
- (B) One-one but not onto
- (C) Not one-one but onto
- (D) Neither one-one nor onto

Q.22 Fundamental period of $f(x) = e^{\cos[x]} + \sin \pi[x]$ is (where $[.]$ and $\{ \}$ denote the greatest integer function and fractional part of function respectively).

- (A) 1
- (B) 2
- (C) π
- (D) 2π

Q.23 If $f(x) = \cos(ax) + \sin(bx)$ is periodic, then which of the followings is false -

- (A) a and b both are rational
- (B) non-periodic if a is rational but b is irrational
- (C) non-periodic if a is irrational but b is rational
- (D) None of these

Q.24 If $f: [-20, 20] \rightarrow \mathbb{R}$ is defined by

$$f(x) = \left[\frac{x^2}{a} \right] \sin x + \cos x, \text{ is an even}$$

function, then the set of values of a is (Where $[.]$ denotes greatest integer function)-

- (A) $(-\infty, 100)$
- (B) $(400, \infty)$
- (C) $(-400, 400)$
- (D) None of these

Q.25 Let f be a function satisfying $f(x+y) = f(x) \cdot f(y)$

for all $x, y \in \mathbb{R}$. If $f(1) = 3$ then $\sum_{r=1}^n f(r)$ is

equal to -

- (A) $\frac{3}{2} (3^n - 1)$
- (B) $\frac{3}{2} n (n + 1)$
- (C) $3^{n+1} - 3$
- (D) None of these

EXERCISE (Level-3)

Part-A : Multiple correct answer type questions

- Q.1** If $f(x) = \sqrt{x^2 - |x|}$, $g(x) = \frac{1}{\sqrt{9 - x^2}}$ then D_{f+g} contains
 (A) $(-3, -1)$ (B) $[1, 3)$
 (C) $[-3, 3]$ (D) $\{0\} \cup [1, 3)$
- Q.2** If $f(x) = \frac{3x-1}{3x^3+2x^2-x}$ and $S = \{x \mid f(x) > 0\}$ then S contains
 (A) $(-\infty, -2)$ (B) $\left(\frac{1}{3}, 5\right)$
 (C) $(-\infty, -1)$ (D) $(0, \infty) - \left\{\frac{1}{3}\right\}$
- Q.3** If D is the domain of the function $f(x) = \sqrt{1-2x} + 3 \sin^{-1}\left(\frac{3x-1}{2}\right)$ then D contains-
 (A) $\left[-\frac{1}{3}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{3}, 0\right]$
 (C) $\left[-\frac{1}{3}, 1\right]$ (D) $\left[\frac{1}{2}, 1\right]$
- Q.4** Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$.
 Let $f: A \rightarrow B$ be defined by $f(x) = \frac{x-3}{x-2}$
 then-
 (A) f is one-one (B) f is onto
 (C) f is bijective (D) None of these
- Q.5** If $F(x) = \frac{\sin \pi[x]}{\{x\}}$, then $F(x)$ is:
 (A) Periodic with fundamental period 1
 (B) Even
 (C) Range is singleton
 (D) Identical to $\operatorname{sgn} \left(\operatorname{sgn} \frac{\{x\}}{\sqrt{\{x\}}} \right) - 1$, where $\{x\}$ denotes fractional part function and $[.]$ denotes greatest integer function and $\operatorname{sgn}(x)$ is a signum function.
- Q.6** Let $f: [-1, 1] \rightarrow [0, 2]$ be a linear function which is onto then $f(x)$ is/are
 (A) $1-x$ (B) $1+x$
 (C) $x-1$ (D) $x+2$
- Q.7** In the following functions defined from $[-1, 1]$ to $[-1, 1]$ the functions which are not bijective are:
 (A) $\sin(\sin^{-1}x)$ (B) $\frac{2}{\pi} \sin^{-1}(\sin x)$
 (C) $(\operatorname{sgn} x) \ln e^x$ (D) $x^3 \operatorname{sgn} x$
- Q.8** Which of the following function is periodic?
 (A) $\operatorname{sgn}(e^{-x})$
 (B) $\sin x + |\sin x|$
 (C) $\min(\sin x, |x|)$
 (D) $\left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$
 Where $[x]$ denotes greatest integer function.
- Q.9** If $f(x) = \begin{cases} 2x+3 & x \leq 1 \\ a^2x+1 & x > 1 \end{cases}$ then values of ' a ' for which $f(x)$ is injective is
 (A) -3 (B) 3
 (C) 0 (D) 1
- Q.10** Consider the function $y = f(x)$ satisfying the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ ($x \neq 0$), then
 (A) domain of $f(x)$ is \mathbb{R}
 (B) domain of $f(x)$ is $\mathbb{R} - (-2, 2)$
 (C) range of $f(x)$ is $[-2, \infty)$
 (D) range of $f(x)$ is $[2, \infty)$
- Q.11** Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = x$. Then
 (A) domain of $f(x)$ is \mathbb{R}
 (B) domain of $f(x)$ is $[-1, 1]$
 (C) range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$
 (D) range of $f(x)$ is \mathbb{R}

- Q.12** Let $f(x) = x^2 - 2ax + a(a+1)$, $f: [a, \infty) \rightarrow [a, \infty)$. If one of the solutions of the equation $f(x) = f^{-1}(x)$ is 5049, then the other may be
 (A) 5051 (B) 5048
 (C) 5052 (D) 5050

- Q.13** If $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x - f(x)$, then $f(17)$ is equal to -
 (A) 17 (B) -51
 (C) 34 (D) -34

- Q.14** $f: \mathbb{R} \rightarrow [-1, \infty)$ and $f(x) = \ell n(|\sin 2x| + |\cos 2x|)$ (where $[\cdot]$ is the greatest integer function)
 (A) $f(x)$ has range \mathbb{Z}
 (B) $f(x)$ is periodic with fundamental period $\pi/4$
 (C) $f(x)$ is invertible in $\left[0, \frac{\pi}{4}\right]$
 (D) $f(x)$ is into function

- Q.15** Let $f(x) = \text{sgn}(\cot^{-1}x) + \tan\left(\frac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal x . Then which of the following alternatives is/are true?
 (A) $f(x)$ is many one but not even function
 (B) $f(x)$ is periodic function
 (C) $f(x)$ is bounded function
 (D) Graph of $f(x)$ remains above the x -axis

- Q.16** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \max(1 + |x|, 2 - |x|)$ then which of the following holds(s) good?
 (A) f is periodic function
 (B) f is neither injective nor surjective
 (C) f is even function
 (D) Range of $f = \left[\frac{3}{2}, \infty\right)$

- Q.17** Which of the following pair(s) of function have same graphs?

(A) $f(x) = \frac{\sec x}{\cos x} - \frac{\tan x}{\cot x}$, $g(x) = \frac{\cos x}{\sec x} + \frac{\sin x}{\csc x}$

(B) $f(x) = \text{sgn}(x^2 - 4x + 5)$,

$g(x) = \text{sgn}\left(\cos^2 x + \sin^2\left(x + \frac{\pi}{3}\right)\right)$ where

sgn denotes signum function

(C) $f(x) = e^{\ell n(x^2 + 3x + 3)}$, $g(x) = x^2 + 3x + 3$

(D) $f(x) = \frac{\sin x}{\sec x} + \frac{\cos x}{\csc x}$, $g(x) = \frac{2\cos^2 x}{\cot x}$

- Q.18** Let f be a constant function with domain \mathbb{R} and g be a certain function with domain \mathbb{R} . Two ordered pairs in f are $(4, a^2 - 5)$ and $(2, 4a - 9)$ for some real number a . Also domain of $\frac{f}{g}$ is $\mathbb{R} - \{7\}$. Then

(A) $a = 2$ (B) $(f(10))^{100} = 1$

(C) $(100)^{g(7)} = 1$ (D) $\int_0^1 f(x) dx = 1$

Part-B : Assertion Reason type Questions

The following questions 19 to 22 consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

- (A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
 (B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.
 (C) If Assertion is true but the Reason is false.
 (D) If Assertion is false but Reason is true

- Q.19** **Assertion** : Function $f(x) = \sin x + \{x\}$ is periodic with period 2π .

Reason : $\sin x$ and $\{x\}$ are both periodic functions with period 2π and 1 respectively.

- Q.20** **Assertion**: If $f(x)$ & $g(x)$ both are one-one, then $f(g(x))$ is also one-one.

Reason : If, $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ then $f(x)$ is one-one

- Q.21** **Assertion**: Let $f: [0, 3] \rightarrow [1, 13]$ is defined by $f(x) = x^2 + x + 1$ then inverse is

$$f^{-1}(x) = \frac{-1 + \sqrt{4x - 3}}{2}$$

Reason: Many-one function is not invertible

- Q.22** **Assertion**: Fundamental period of $\cos x + \cot x$ is 2π .

Reason: If the period of $f(x)$ is T_1 and the period of $g(x)$ is T_2 , then the fundamental period of $f(x) + g(x)$ is the L.C.M. of T_1 and T_2 .

Part-C : Column Matching type Questions

Match the entry in Column 1 with the entry in Column 2.

Q.23 Match the following :

Column 1

Column 2

(A) Range of $\sqrt{[\sin 2x] - [\cos 2x]}$ (P) {1, 2, 3}

(B) Domain of $\sqrt{x^2 + 4x} C_{2x^2+3}$ (Q) {1}

(C) Range of $\sqrt{\log(\cos(\sin x))}$ (R) {0, 1}

(D) Range of $[|\sin x| + |\cos x|]$ (S) {0}
(Where $[.]$ denotes greatest integer function)

Q.24 Match the following :

Column 1

Column 2

(A) Period of $\frac{1}{2} \left\{ \frac{|\sin x|}{\cos x} + \frac{|\cos x|}{\sin x} \right\}$ (P) 2

(B) Range of $\cos^{-1} \sqrt{\log_{[x]} \frac{|x|}{x}}$ (Q) 2π

(C) Total number of solution of $x^2 - 4 - [x] = 0$ is (R) 1

(D) Fundamental period of $e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$ (S) $\frac{\pi}{2}$

(Where $[.]$ denotes greatest integer function)

Q.25 Match the following :

Column 1

Column 2

(A) Domain of $f(x) = \sqrt{2^x - 3^x} + \log_3 \log_{1/2} x$ is (P) [0, 1]

(B) Solution set of equation $2\cos^2 \frac{x}{2} \sin^2 x = x^2 + \frac{1}{x^2}$ is (Q) [0, ∞)

(C) If $A = \left\{ (x, y); y = \frac{1}{x}, x \in R_0 \right\}$ & $B = \{(x, y); y = -x, x \in R\}$
then $A \cap B$ is (R) [1, ∞)

(D) The functions $f(x) = \sqrt{x} \sqrt{x-1}$ & $\phi(x) = \sqrt{x^2 - x}$ are identical in (S) ϕ

Q.26 Match the following :

Column 1

Column 2

(A) The fundamental period of the function (P) $\frac{1}{2}$

$$y = \sin\left(2\pi t + \frac{\pi}{3}\right) + 2 \sin\left(3\pi t + \frac{\pi}{4}\right) + 3 \sin 5\pi t$$

(B) $y = \{\sin(\pi x)\}$ is a many one function for $x \in (0, a)$ where $\{ \}$ denotes fractional part of x and a may be (Q) 8

(C) The Fundamental period of the function (R) 2

$$y = \frac{1}{2} \left(\frac{|\sin(\pi/4)x|}{\cos(\pi/4)x} + \frac{\sin(\pi/4)x}{|\cos(\pi/4)x|} \right)$$

(D) If $f: [0, 2] \rightarrow [0, 2]$ is bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non-zero real numbers, then $f(2)$ is equal to (S) 0

Part-D : Passage based objective questions

Passage # 1 (Q.27 to 29)

Let $f(x) = x^2 - 3x + 2$, $g(x) = f(|x|)$

$h(x) = |g(x)|$ and $I(x) = |g(x)| - [x]$

are four function, where $[x]$ is the integral part of real x .

Q.27 Find the value of 'a' such that equation $g(x) - a = 0$ has exactly 3 real roots-

- (A) 2 (B) 1
(C) 0 (D) None of these

Q.28 Find the set of values of 'b' such that equation $h(x) - b = 0$ has exactly 8 real solution

- (A) $b \in \left[0, \frac{1}{4}\right]$ (B) $b \in \left[0, \frac{1}{4}\right)$
(C) $b \in \left(0, \frac{1}{4}\right)$ (D) None of these

- Q.29** Which statement is true for $I(x) = 0$ -
 (A) Two values of x is satisfied for $I(x) = 0$
 (B) One value of x is satisfied for $I(x) = 0$ and that x lie between 1 and 2
 (C) One value of x is satisfied for $I(x) = 0$ and that x lie between 3 and 4
 (D) None of these

Passage # 2 (Q.30 to 32)

If $f(x) = 0$; if $x \in \mathbb{Q}$
 $= 1$; if $x \notin \mathbb{Q}$.

then answer the following questions-

- Q.30** $f(x)$ is -
 (A) an even function
 (B) an odd function
 (C) Neither even nor odd function
 (D) one-one function
- Q.31** $f(f(x))$ is-
 (A) a constant function
 (B) an even function
 (C) an odd function
 (D) many one function
- Q.32** Domain of $g(x) = \ln(\operatorname{sgn} f(x))$ is-
 (A) \mathbb{R}
 (B) set of all rational numbers
 (C) set of all irrational number
 (D) \mathbb{R}^+

Passage # 3 (Q.33 to 35)

Consider the function

$$f(x) = \begin{cases} x - [x] - \frac{1}{2} & \text{if } x \notin \mathbb{I} \\ 0 & \text{if } x \in \mathbb{I} \end{cases}$$

where $[.]$ denotes greatest integer function.

If $g(x) = \max.\{x^2, f(x), |x|\}$; $-2 \leq x \leq 2$, then

- Q.33** Range of $f(x)$ is-
 (A) $[0, 1)$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
 (C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right)$

- Q.34** $f(x)$ is-
 (A) non periodic
 (B) periodic with fundamental period 1
 (C) periodic with fundamental period 2
 (D) periodic with fundamental period $\frac{1}{2}$

- Q.35** The set of values of a , if $g(x) = a$ has three real and distinct solutions, is -

- (A) $\left(0, \frac{1}{2}\right)$ (B) $\left(0, \frac{1}{4}\right)$
 (C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) $(0, 1)$

Passage # 4 (Q.36 to 38)

Consider the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x \leq 1 \\ \ln x, & 1 < x \leq e \end{cases}$$

Let $f_1(x) = f(|x|)$

$$f_2(x) = |f(|x|)|$$

$$f_3(x) = f(-x)$$

- Q.36** Number of positive solutions of the equation $2f_2(x) - 1 = 0$ is-
 (A) 4 (B) 3
 (C) 2 (D) 1
- Q.37** Number of integral solution of the equation $f_1(x) = f_2(x)$ is
 (A) 1 (B) 2
 (C) 3 (D) 4
- Q.38** If $f_4(x) = \log_{27}(f_3(x) + 2)$, then range of $f_4(x)$ is
 (A) $[1, 9]$ (B) $\left[\frac{1}{3}, \infty\right)$
 (C) $\left[0, \frac{1}{3}\right]$ (D) $[1, 27]$

Part-E : Numeric Response Type Questions

- Q.39** Let $f(x) = \left[\frac{1}{\cos\{x\}}\right]$ where $[y]$ and $\{y\}$ denote greatest integer and fractional part function respectively and $g(x) = 2x^2 - 3x(k+1) + k(3k+1)$. If $g(f(x)) < 0 \forall x \in \mathbb{R}$ then find the number of integral values of k .

Q.40 If $x = \log_4 \left(\frac{2f(x)}{1-f(x)} \right)$, then find $(f(2010) + f(-2009))$.

Q.41 If M and m are maximum and minimum value of $f(\theta) = 5 \sin^2 \theta - 8 \sin \theta + 4$, $\theta \in \left[\frac{\pi}{3}, \frac{5\pi}{6} \right]$, respectively then find the value of $(2Mm)$.

Q.42 Let f be a function such that $4 f(x^{-1} + 1) + 8 f(x + 1) = \log_{12} x$, then find the value of $4 (f(10) + f(13) + f(17))$.

Q.43 Let f be a real valued invertible function such that $f \left(\frac{2x-3}{x-2} \right) = 5x - 2$, $x \neq 2$. Find $f^{-1}(13)$.

Part-F : Subjective Type Questions

Q.44 Find the domains of definitions of the following functions:
(Read the symbols $[*]$ and $\{*\}$ as greatest integers and fractional part functions respectively)

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16-x^2}$

(ii) $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x)$

(iii) $f(x) = \ell n \left(\sqrt{x^2 - 5x - 24} - x - 2 \right)$

(iv) $f(x) = \sqrt{\frac{1-5^x}{7^{-x}-7}}$

(v) $y = \log_{10} \sin(x-3) + \sqrt{16-x^2}$

(vi) $f(x) = \log_{100x} \left(\frac{2 \log_{10} x + 1}{-x} \right)$

(vii) $f(x) = \frac{1}{\sqrt{4x^2-1}} + \ell n x(x^2-1)$

(viii) $f(x) = \sqrt{\log_{1/2} \frac{x}{x^2-1}}$

(ix) $f(x) = \sqrt{x^2 - |x|} + \frac{1}{\sqrt{9-x^2}}$

(x) $f(x) = \sqrt{\log_x (\cos 2\pi x)}$

(xi) $f(x) = \frac{\sqrt{\cos x - (1/2)}}{\sqrt{6+35x-6x^2}}$

(xii) $f(x) = \sqrt{\log_{1/3} (\log_4 ([x]^2 - 5))}$

(xiii) $f(x) = \frac{[x]}{2x - [x]}$

(xiv) $f(x) = \log_x \sin x$

(xv) $f(x) = \log_{\left[x+\frac{1}{x}\right]} |x^2 - x - 6| +$

$16^{-x} C_{2x-1} + 20^{-3x} P_{2x-5}$

Q.45 Find the domain and range of the following functions. (Read the symbols $[*]$ & $\{*\}$ as greatest integers & fractional part functions respectively.)

(i) $y = \log_{\sqrt{5}} (\sqrt{2}(\sin x - \cos x) + 3)$

(ii) $y = \frac{2x}{1+x^2}$

(iii) $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$

(iv) $f(x) = \frac{x}{1+|x|}$

(v) $y = \sqrt{2-x} + \sqrt{1+x}$

(vi) $f(x) = \log_{(\csc x - 1)} (2 - [\sin x] - [\sin x]^2)$

(vii) $f(x) = \frac{\sqrt{x+4} - 3}{x-5}$

(viii) $\cot^{-1} (2x - x^2)$.

(ix) $f(x) = \log_2 (\sqrt{x-4} + \sqrt{6-x})$

Q.46 (a) Draw graphs of the following function, where $[]$ denotes the greatest integer function.

(i) $f(x) = x + [x]$

(ii) $y = (x)[x]$ where $x = [x] + \{x\}$
& $x > 0$ and $x \leq 3$

(iii) $y = \operatorname{sgn} [x]$

(iv) $\operatorname{sgn} (x - |x|)$

(b) Identify the pair(s) of functions which are identical? (where $[x]$ denotes greatest integer and $\{x\}$ denotes fractional part function)

(i) $f(x) = \operatorname{sgn} (x^2 - 3x + 4)$ and $g(x) = e^{\{x\}}$

(ii) $f(x) = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ and $g(x) = \tan x$

(iii) $f(x) = \ell n(1+x) + \ell n(1-x)$ and $g(x) = \ell n(1-x^2)$

(iv) $f(x) = \frac{\cos x}{1-\sin x}$ & $g(x) = \frac{1+\sin x}{\cos x}$

EXERCISE (Level-4)

Old Examination Questions

Section-A [JEE Main]

- Q.1** Let $f : (-1, 1) \rightarrow \mathbb{R}$, be a function defined by $f(x) = \tan^{-1} \frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

[AIEEE-2005]

- (A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left[0, \frac{\pi}{2}\right)$
(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

- Q.2** A real valued function $f(x)$ satisfies the functional equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ where a is a given constant and $f(0) = 1$, then $f(2a-x)$ is equal to -

[AIEEE-2005]

- (A) $-f(x)$ (B) $f(x)$
(C) $f(a) + f(a-x)$ (D) $f(-x)$

- Q.3** The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ defined, is-

[AIEEE 2007]

- (A) $[0, \pi]$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(C) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left[0, \frac{\pi}{2}\right)$

- Q.4** Let $f : \mathbb{N} \rightarrow \mathbb{Y}$ be a function defined as $f(x) = 4x + 3$ where $\mathbb{Y} = \{y \in \mathbb{N} : y = 4x + 3 \text{ for some } x \in \mathbb{N}\}$. Inverse of f is -

[AIEEE 2008]

- (A) $g(y) = 4 + \frac{y+3}{4}$ (B) $g(y) = \frac{y+3}{4}$
(C) $g(y) = \frac{y-3}{4}$ (D) $g(y) = \frac{3y+4}{3}$

- Q.5** For real x , let $f(x) = x^3 + 5x + 1$, then -

[AIEEE 2009]

- (A) f is one - one but not onto on \mathbb{R}
(B) f is onto on \mathbb{R} but not one - one
(C) f is one - one and onto on \mathbb{R}
(D) f is neither one - one nor onto on \mathbb{R}

- Q.6** Let $f(x) = (x+1)^2 - 1$, $x \geq -1$

Statement - 1 :

The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

Statement - 2 :

f is a bijection.

[AIEEE 2009]

- (A) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement -1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement -1.
(C) Statement -1 is true, Statement-2 is false.
(D) Statement -1 is false, Statement-2 is true

- Q.7** The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$ is : [AIEEE 2011]

- (A) $(-\infty, \infty)$ (B) $(0, \infty)$
(C) $(-\infty, 0)$ (D) $(-\infty, \infty) - \{0\}$

- Q.8** Let A and B be nonempty set in \mathbb{R} and $f : A \rightarrow B$ be a bijective function.

Statement-1: f is an onto function

Statement-2 : There exists a function

$g : B \rightarrow A$ such that $f \circ g = I_B$.

[AIEEE Online- 2012]

- (A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1
(B) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true

Section-B [JEE Advanced]

Q.1 $f(x) = \begin{cases} x, & x \in Q \\ 0, & x \notin Q \end{cases}; g(x) = \begin{cases} 0 & x \in Q \\ x & x \notin Q \end{cases}$

then $(f - g)$ is [IIT Scr. 2005]

- (A) one-one, onto
(B) neither one-one, nor onto
(C) one-one but not onto
(D) onto but not one-one

Q.2 If X and Y are two non-empty sets where $f: X \rightarrow Y$ is function is defined such that $f(C) = \{f(x): x \in C\}$ for $C \subseteq X$

and $f^{-1}(D) = \{x: f(x) \in D\}$ for $D \subseteq Y$ for any $A \subseteq X$ and $B \subseteq Y$ then- [IIT 2005]

- (A) $f^{-1}(f(A)) = A$
(B) $f^{-1}(f(A)) = A$ only if $f(X) = Y$
(C) $f(f^{-1}(B)) = B$ only if $B \subseteq f(X)$
(D) $f(f^{-1}(B)) = B$

Q.3 Find the range of values of t for which

$$2 \sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}; t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

[IIT 2005]

Q.4 Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$ [IIT 2007].

Column-I

Column-II

- (A) If $-1 < x < 1$, then $f(x)$ satisfies
(B) If $1 < x < 2$, then $f(x)$ satisfies
(C) If $3 < x < 5$, then $f(x)$ satisfies
(D) If $x > 5$, then $f(x)$ satisfies
- (P) $0 < f(x) < 1$
(Q) $f(x) < 0$
(R) $f(x) > 0$
(S) $f(x) < 1$

Q.5 Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is [IIT 2011]

- (A) $\pm \sqrt{n\pi}, n \in \{0, 1, 2, \dots\}$
(B) $\pm \sqrt{n\pi}, n \in \{1, 2, \dots\}$
(C) $\frac{\pi}{2} + 2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$
(D) $2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

Q.6 The function $f: [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is [IIT 2012]

- (A) one-one and onto.
(B) onto but not one-one.
(C) one-one but not onto.
(D) neither one-one nor onto.

Q.7 Let $f: (-1, 1) \rightarrow \mathbb{R}$ be such that

$$f(\cos 4\theta) = \frac{2}{2 - \sec^2 \theta} \text{ for } \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right).$$

Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

MCQ [IIT 2012]

- (A) $1 - \sqrt{\frac{3}{2}}$ (B) $1 + \sqrt{\frac{3}{2}}$
(C) $1 - \sqrt{\frac{2}{3}}$ (D) $1 + \sqrt{\frac{2}{3}}$

Q.8 Let $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$ be given by

$$f(x) = (\log(\sec x + \tan x))^3. \text{ Then}$$

MCQ [IIT-Advance 2014]

- (A) $f(x)$ is an odd function
(B) $f(x)$ is a one-one function
(C) $f(x)$ is an onto function
(D) $f(x)$ is an even function

Q.9 Let $f(x) = \sin\left(\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)\right)$ for all $x \in \mathbb{R}$

$$\text{and } g(x) = \frac{\pi}{2} \sin x \text{ for all } x \in \mathbb{R}. \text{ Let } (f \circ g)(x)$$

denote $f(g(x))$ and $(g \circ f)(x)$ denote $g(f(x))$. Then which of the following is (are) true?

MCQ [IIT-Advance 2015]

(A) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) Range of $f \circ g$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is an $x \in \mathbb{R}$ such that $(g \circ f)(x) = 1$

- Q.10** Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X , then the value of $\frac{1}{5!} (\beta - \alpha)$ is _____ . **[JEE - Advance 2018]**

- Q.11** Let $E_1 = \left\{ x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0 \right\}$ and $E_2 = \left\{ x \in E_1 : \sin^{-1} \left(\log_e \left(\frac{x}{x-1} \right) \right) \text{ is a real number} \right\}$
- $\left(\begin{array}{l} \text{Here, the inverse trigonometric function} \\ \sin^{-1} x \text{ assumes values in } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]. \end{array} \right)$
- Let $f : E_1 \rightarrow \mathbb{R}$ be the function defined by $f(x) = \log_e \left(\frac{x}{x-1} \right)$ and $g : E_2 \rightarrow \mathbb{R}$ be the function defined by $g(x) = \sin^{-1} \left(\log_e \left(\frac{x}{x-1} \right) \right)$
- [JEE - Advance 2018]**

List-I

- (P) The range of f is
 (Q) The range of g contains
 (R) The domain of f contains
 (S) The domain of g is

List-II

- (1) $\left(-\infty, \frac{1}{1-e} \right] \cup \left[\frac{e}{e-1}, \infty \right)$
 (2) $(0, 1)$
 (3) $\left[-\frac{1}{2}, \frac{1}{2} \right]$
 (4) $(-\infty, 0) \cup (0, \infty)$
 (5) $\left(-\infty, \frac{e}{e-1} \right]$
 (6) $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1} \right]$

The correct option is :

- (A) $P \rightarrow 4 ; Q \rightarrow 2 ; R \rightarrow 1 ; S \rightarrow 1$
 (B) $P \rightarrow 3 ; Q \rightarrow 3 ; R \rightarrow 6 ; S \rightarrow 5$
 (C) $P \rightarrow 4 ; Q \rightarrow 2 ; R \rightarrow 1 ; S \rightarrow 6$
 (D) $P \rightarrow 4 ; Q \rightarrow 3 ; R \rightarrow 6 ; S \rightarrow 5$

EXERCISE (Level-5)

Review Exercise

- Q.1** Find the natural number 'a' for which $\sum_{k=1}^n f(a+k) = 16(2^n - 1)$, where the function 'f' satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x, y and further $f(1) = 2$. [IIT-1992]
- Q.2** A function $f: \mathbb{R} \rightarrow \mathbb{R}$, where \mathbb{R} , is the set of real numbers, is defined by $f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Find the interval of values of α for which $f(x)$ is onto. Is the functions one-to-one for $\alpha = 3$? Justify your answer. [IIT 1996]
- Q.3** Let $f(x) = [x] \sin\left(\frac{\pi}{[x+1]}\right)$, where $[.]$ denotes the greatest integer function. Then find the domain of f . [IIT 1996]
- Q.4** If f is an even function defined on the interval $(-5, 5)$, then four real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are and. [IIT-1996]
- Q.5** If the function $f: [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then find the value of $f^{-1}(x)$. [IIT 99]
- Q.6** Let $[x]$ = the greatest integer less than or equal to x . If all the values of x such that the product $\left[x - \frac{1}{2}\right] \left[x + \frac{1}{2}\right]$ is prime, belongs to the set $[x_1, x_2) \cup [x_3, x_4)$, find the value of $x_1^2 + x_2^2 + x_3^2 + x_4^2$.
- Q.7** The set of real values of 'x' satisfying the equality $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right] = 5$ (where $[.]$ denotes the greatest integer function) belongs to the interval $\left(a, \frac{b}{c}\right]$ where $a, b, c \in \mathbb{N}$ and $\frac{b}{c}$ is in its lowest form. Find the value of $a + b + c + abc$.
- Q.8** Let $f: \mathbb{R} \rightarrow \mathbb{R} - \{3\}$ be a function with the property that there exist $T > 0$ such that $f(x+T) = \frac{f(x)-5}{f(x)-3}$ for every $x \in \mathbb{R}$. Prove that $f(x)$ is periodic.
- Q.9** In a function $2f(x) + xf\left(\frac{1}{x}\right) - 2f\left(\sqrt{2} \sin\left(\pi\left(x + \frac{1}{4}\right)\right)\right) = 4 \cos^2 \frac{\pi x}{2} + x \cos \frac{\pi}{x}$. Prove that (i) $f(2) + f\left(\frac{1}{2}\right) = 1$ (ii) $f(2) + f(1) = 0$
- Q.10** Verify if $f(x) = \frac{x^2 - 8x + 18}{x^2 + 4x + 30}$ is an one-one function.
- Q.11** Find the domain of the function, $f(x) = \frac{1}{[|x-1|] + [|7-x|] - 6}$ Where $[.]$ is greatest integer function.
- Q.12** Let n be a positive integer and define $f(n) = 1! + 2! + 3! + \dots + n!$, where $n! = n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$. Find the polynomial $P(x)$ and $Q(x)$ such that $f(n+2) = P(n)f(n+1) + Q(n)f(n)$, for all $n \geq 1$.
- Q.13** Find the domain of $y = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2 \frac{2x-1}{3+x}\right)}$

ANSWER KEY

EXERCISE (Level-1)

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (A) | 2. (D) | 3. (C) | 4. (D) | 5. (A) | 6. (C) | 7. (B) |
| 8. (C) | 9. (C) | 10. (B) | 11. (C) | 12. (A) | 13. (D) | 14. (C) |
| 15. (A) | 16. (D) | 17. (B) | 18. (D) | 19. (B) | 20. (A) | 21. (B) |
| 22. (C) | 23. (B) | 24. (C) | 25. (A) | 26. (A) | | |

EXERCISE (Level-2)

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (C) | 2. (B) | 3. (B) | 4. (B) | 5. (C) | 6. (B) | 7. (D) |
| 8. (D) | 9. (C) | 10. (A) | 11. (C) | 12. (D) | 13. (B) | 14. (B) |
| 15. (D) | 16. (C) | 17. (D) | 18. (C) | 19. (B) | 20. (B) | 21. (D) |
| 22. (A) | 23. (D) | 24. (B) | 25. (A) | | | |

EXERCISE (Level-3)

Part-A

- | | | | | | | |
|---------------|--------------|---------------|-------------|--------------|----------|------------|
| 1. (A,B,D) | 2. (A,B,C,D) | 3. (A,B) | 4. (A,B,C) | 5. (A,B,C,D) | 6. (A,B) | 7. (B,C,D) |
| 8. (A,B,C,D) | 9. (A,B) | 10. (B,D) | 11. (B,C) | 12. (B,D) | 13. (C) | 14. (B,D) |
| 15. (A,B,C,D) | 16. (B,C,D) | 17. (A,B,C,D) | 18. (A,B,C) | | | |

Part-B

- | | | | |
|---------|---------|---------|---------|
| 19. (D) | 20. (A) | 21. (B) | 22. (C) |
|---------|---------|---------|---------|

Part-C

- | | |
|--|--|
| 23. $A \rightarrow R, B \rightarrow P, C \rightarrow S, D \rightarrow Q$ | 24. $A \rightarrow Q, B \rightarrow S, C \rightarrow P, D \rightarrow R$ |
| 25. $A \rightarrow S, B \rightarrow S, C \rightarrow S, D \rightarrow R$ | 26. $A \rightarrow R, B \rightarrow Q,R, C \rightarrow Q, D \rightarrow S$ |

Part-D

- | | | | | | | |
|---------|---------|---------|---------|---------------|---------|---------|
| 27. (A) | 28. (C) | 29. (C) | 30. (A) | 31. (A,B,C,D) | 32. (C) | 33. (C) |
| 34. (B) | 35. (C) | 36. (C) | 37. (D) | 38. (C) | | |

Part-E

- | | | | | |
|-------|-------|-------|-------|-------|
| 39. 1 | 40. 1 | 41. 2 | 42. 3 | 43. 3 |
|-------|-------|-------|-------|-------|

Part-F

44. (i) $\left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ (ii) $\left(-4, -\frac{1}{2}\right) \cup (2, \infty)$ (iii) $(-\infty, -3]$ (iii) $(-\infty, -3]$
- (iv) $(-\infty, -1) \cup [0, \infty)$ (v) $(3 - 2\pi < x < 3 - \pi) \cup (3 < x \leq 4)$
- (vi) $\left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}}\right)$ (vii) $(-1 < x < -1/2) \cup (x > 1)$ (vii) $\left[\frac{1-\sqrt{5}}{2}, 0\right] \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$
- (ix) $(-3, -1] \cup \{0\} \cup [1, 3)$ (x) $\left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right) \cup \{x : x \in \mathbb{N}, x \geq 2\}$ (xi) $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$
- (xii) $[-3, -2) \cup [3, 4)$ (xiii) $\mathbb{R} - \left\{-\frac{1}{2}, 0\right\}$
- (xiv) $2n\pi < x < (2n+1)\pi$ but $x \neq 1$ where n is non-negative integer. (xv) $x \in \{4, 5\}$
45. (i) $D : x \in \mathbb{R} \quad R : [0, 2]$ (ii) $D = \mathbb{R} ; \text{range } [-1, 1]$
- (iii) $D : \{x \mid x \in \mathbb{R}; x \neq -3; x \neq 2\} \quad R : \{f(x) \mid f(x) \in \mathbb{R}, f(x) \neq 1/5; f(x) \neq 1\}$
- (iv) $D : \mathbb{R} ; R : (-1, 1)$ (v) $D : -1 \leq x \leq 2; R : [\sqrt{3}, \sqrt{6}]$
- (vi) $D : x \in (2n\pi, (2n+1)\pi) - \{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in \mathbb{I}\}$ and Range is $(-\infty, \infty) - \{0\}$
- (vii) $D : [-4, \infty) - \{5\}; R : \left(0, \frac{1}{6}\right) \cup \left(\frac{1}{6}, \frac{1}{3}\right]$ (viii) $\left[\frac{\pi}{4}, \pi\right)$ (ix) $\left[\frac{1}{2}, 1\right]$
46. (b) (i), (iii) are identical. 48. $f(x) = \frac{x+1}{x-1}$ 49. $\left(0, \frac{5}{3}\right)$ 50. $[0, 4)$

EXERCISE (Level-4)

SECTION-A

1. (D) 2. (A) 3. (D) 4. (C) 5. (C) 6. (B) 7. (C)
8. (A)

SECTION-B

1. (A) 2. (C) 3. $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ 4. $A \rightarrow P, R, S; B \rightarrow Q, S; C \rightarrow Q, S; D \rightarrow P, R, S$
5. (A) 6. (B) 7. (This question was awarded as bonus) 8. (A, B, C) 9. (A, B, C)
10. 119.00 11. (A)

EXERCISE (Level-5)

1. $a = 3$ 2. $\alpha \in \phi$ 3. $\{x \in \mathbb{R} \mid x \notin [-1, 0)\}$ 4. $\frac{\pm 3 \pm \sqrt{5}}{2}$ 5. $\frac{1}{2} \left(1 + \sqrt{1 + 4 \log_2 x}\right)$
6. 11 7. 20 10. No 11. $\mathbb{R} - (0, 1) \cup \{1, 2, 3, 4, 5, 6, 7\} \cup (7, 8)$
12. $P(x) = x + 3, Q(x) = -(x + 2)$ 13. $(-4, -3) \cup (4, \infty)$